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Delay-Dependent Robust Stability Criteria for Uncertain Neutral Systems with Mixed Delays

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Abstract

This paper concerns the problem of the delay-dependent robust stability of neutral systems with mixed delays and time-varying structured uncertainties. A new method based on linear matrix inequalities is presented that makes it easy to calculate both the upper stability bounds on the delays and the free weighting matrices. Since the criteria take the sizes of the neutral- and discrete-delays into account, it is less conservative than previous methods. Numerical examples illustrate both the improvement this approach provides over previous methods and the reciprocal influences between the neutral- and discrete-delays.

Keywords: neutral system, delay-dependent criteria, robust stability, time-varying structured uncertainties, linear matrix inequality.

1 Introduction

There are two types of time delay systems: retarded and neutral. The retarded type ([4]) contains delays only in its states (this kind of delay is called a discrete delay hereafter), whereas the neutral type contains delays both in its states and in the derivatives of its states. Neutral-type delay systems can be found in such places as population ecology, distributed networks containing lossless transmission lines, heat exchangers, robots in contact with rigid environments, etc (e.g., [14], [16], [20]). Some new control technologies, like repetitive control, use the neutral type through the insertion of an artificial neutral delay into a control loop in order to boost the control performance for periodic signals ([10]). While the number of unstable poles is finite in a retarded-delay system, it is infinite in a neutral-delay system. That makes a neutral-delay system much harder to stabilize, and a considerable number of studies have been carried out on the stability problem of the neutral type.

Although frequency-domain methods, such as the use of the Nyquist stability criterion, are very effective in the analysis and synthesis of a control system, they cannot easily handle a delay system that contains time-varying structured uncertainties (see [2] and [7]). Instead, the Lyapunov-functional approach is widely employed because it is known to be effective; and linear

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matrix inequalities (LMIs) provide a powerful and efficient numerical tool for the calculations. The stability criteria thus obtained can be divided into two categories: delay-independent ([2], [11], [19]) and delay-dependent ([3], [5], [8], [9], [13], [17], [18], [21], [22], [23]). The delay-independent type is independent of delay size; and as might be expected, it is generally conservative, especially when a delay is short. On the other hand, studies of delay-dependent criteria have focused mainly on identical delays in neutral and discrete terms ([3], [5], [9], [17], [18], [22]). Some papers have also presented criteria that depend only on the size of discrete delays, and not on the size of neutral delays ([8], [13], [21], [23]). They are called discrete-delay-dependent and neutral-delay-independent stability criteria ([8]). Although a great deal of effort has been devoted to the investigation of this subject, our knowlegde of neutral- and discrete-delay-dependent stability criteria is still insufficient.

Recently, Park ([24]) presented a new method of obtaining a delay-dependent criterion for a discrete-delay system, which was less conservative than previous methods. It was later extended to a neutral-delay system ([22], [23]). However, that analysis used the Leibniz-Newton formula in the derivative of a Lyapunov-Krasovskii functional, and replaced the term $x(t-\tau)$ with $x(t)-\int_{t-\tau}^{t}\dot{x}(s)ds$ in some places, but not in others, in order to make the Lyapunov-Krasovskii functional easier to handle. Clearly there must be a relationship between $x(t-\tau)$ and $x(t)-\int_{t-\tau}^{t}\dot{x}(s)ds$ because both of them affect the result; but it was not taken into account. In addition, even though the free weighting matrices for those terms are very important, no method of selecting them was given.

This paper presents a new method of dealing with the problem of the delay-dependent stability of neutral systems. First, a criterion for a nominal neutral system is derived. This method employs free weighting matrices to express the influences of, and the relationship between, the terms $x(t-\tau)$ and $x(t)-\int_{t-\tau}^{t}\dot{x}(s)ds$. The new criterion is based on LMIs, which makes the free weighting matrices easy to select. Since this criterion is both neutral-delay-dependent and discrete-delay-dependent, it is less conservative than previous methods for mixed neutral- and discrete-delays. The criterion obtained is then extend to a neutral system with time-varying uncertainties. Finally, some numerical examples illustrate both the improvement the proposed approach provides over previous methods and also the reciprocal influences between neutral and discrete delays.

2 Notation and Preliminaries

Consider the time-varying structured uncertain neutral system

$$\Sigma: \begin{cases} \dot{x}(t)-C\dot{x}(t-\tau_2)=(A+\Delta A(t))x(t)+(B+\Delta B(t))x(t-\tau_1),\ t>0, \\ x(t)=\phi(t),\ t\in[-\tau,\,0], \end{cases} \tag{1}$$

where $x(t)\in R^n$ is the state vector; $\tau_1,\,\tau_2>0$ are constant delays; $\tau:=max(\tau_1,\,\tau_2)$, $A,\,B,\,C\in R^{n\times n}$ are constant matrices; and the spectrum radius of the matrix C, $\rho(C)$, satisfies $\rho(C)<1$. The initial condition $\phi(t)$ denotes a continuous vector-valued initial function of $t\in[-\tau,\,0]$. The time-varying structured uncertainties are of the form

$$[\Delta A(t)\ \Delta B(t)]=DF(t)\,[E_a\ E_b]\,, \tag{2}$$

where $D, E_a,\, E_b$ are constant matrices with appropriate dimensions; and $F(t)$ is an unknown, real, and possibly time-varying matrix with Lsbesgue measurable elements, and its Euclidean norm satisfies

$$\|F(t)\|\le 1,\ \forall t. \tag{3}$$

First, the nominal system Σ_0 of Σ is defined to be

$$\Sigma_0 : \begin{cases} \dot{x}(t) - C\dot{x}(t-\tau_2) = Ax(t) + Bx(t-\tau_1),\ t > 0, \\ x(t) = \phi(t),\ t \in [-\tau,\ 0]. \end{cases} \tag{4}$$

To obtain the main results, the following lemma is necessary to deal with the uncertainties.

Lemma 1 [25] *Given matrices $Q = Q^T$, H, E and $R = R^T > 0$ of appropriate dimensions, then*

$$Q + HFE + E^T F^T H^T < 0,$$

for all F satisfying $F^T F \leq R$, if and only if there exists an $\varepsilon > 0$ such that

$$Q + \varepsilon^2 HH^T + \varepsilon^{-2} E^T R E^T < 0.$$

3 Main Results

In order to simplify the treatment of the problem, the operator $\mathcal{D}$: $C([-\tau_2, 0], R^n) \to R^n$ is first defined to be

$$\mathcal{D}x_t = x(t) - Cx(t-\tau_2).$$

The stability of $\mathcal{D}$ is defined as follows:

Definition 1 *([6]) The operator $\mathcal{D}$ is said to be stable if the zero solution of the homogeneous difference equation*

$$\mathcal{D}x_t = 0,\ t \geq 0,\ x_0 = \psi \in \{\phi \in C([-\tau_2,\ 0] :\ \mathcal{D}\phi = 0\}$$

is uniformly asymptotically stable.

The necessary condition for the stability of Σ and Σ_0 is that the operator $\mathcal{D}$ be stable ([6]). The following theorem governs the nominal system Σ_0.

Theorem 1 *Given scalars $\tau_1 > 0$ and $\tau_2 > 0$, the nominal system Σ_0 is asymptotically stable if the operator $\mathcal{D}$ is stable and there exist positive definite matrices $P = P^T > 0$, $Q_i = Q_i^T > 0\ (i = 1,\ 2)$ and $R = R^T > 0$, non-negative definite matrices $X_{ii} = X_{ii}^T \geq 0$ and $Y_{ii} = Y_{ii}^T \geq 0\ (i = 1,\ \cdots,\ 5)$ and any matrices X_{ij} and Y_{ij} $(i = 1,\ \cdots,\ 5;\ i < j \leq 5)$ such that the following LMIs are feasible.*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & A^T S \\ \Phi_{12}^T & \Phi_{22} & \Phi_{23} & \Phi_{24} & B^T S \\ \Phi_{13}^T & \Phi_{23}^T & \Phi_{33} & \Phi_{34} & 0 \\ \Phi_{14}^T & \Phi_{24}^T & \Phi_{34}^T & \Phi_{44} & C^T S \\ SA & SB & 0 & SC & -S \end{bmatrix} < 0, \tag{5}$$

$$\Psi = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ X_{12}^T & X_{22} & X_{23} & X_{24} & X_{25} \\ X_{13}^T & X_{23}^T & X_{33} & X_{34} & X_{35} \\ X_{14}^T & X_{24}^T & X_{34}^T & X_{44} & X_{45} \\ X_{15}^T & X_{25}^T & X_{35}^T & X_{45}^T & X_{55} \end{bmatrix} \geq 0, \tag{6}$$

$$\Xi = \begin{bmatrix} Y_{11} & Y_{12} & X_{13} & Y_{14} & Y_{15} \\ Y_{12}^T & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{13}^T & Y_{23}^T & Y_{33} & Y_{34} & Y_{35} \\ Y_{14}^T & Y_{24}^T & Y_{34}^T & Y_{44} & Y_{45} \\ Y_{15}^T & Y_{25}^T & Y_{35}^T & Y_{45}^T & Y_{55} \end{bmatrix} \geq 0, \tag{7}$$

where

$$\begin{cases} \Phi_{11} = PA + A^T P + Q_1 + Q_2 + X_{15} + X_{15}^T + Y_{15} + Y_{15}^T + \tau_1 X_{11} + \tau_2 Y_{11}, \\ \Phi_{12} = PB - X_{15} + X_{25}^T + Y_{25}^T + \tau_1 X_{12} + \tau_2 Y_{12}, \\ \Phi_{13} = -A^T PC + X_{35}^T + Y_{35}^T - Y_{15} + \tau_1 X_{13} + \tau_2 Y_{13}, \\ \Phi_{14} = X_{45}^T + Y_{45}^T + \tau_1 X_{14} + \tau_2 Y_{14}, \\ \Phi_{22} = -Q_1 - X_{25} - X_{25}^T + \tau_1 X_{22} + \tau_2 Y_{22}, \\ \Phi_{23} = -B^T PC - X_{35}^T - Y_{25} + \tau_1 X_{23} + \tau_2 Y_{23}, \\ \Phi_{24} = -X_{45}^T + \tau_1 X_{24} + \tau_2 Y_{24}, \\ \Phi_{33} = -Q_2 - Y_{35} - Y_{35}^T + \tau_1 X_{33} + \tau_2 Y_{33}, \\ \Phi_{34} = -Y_{45}^T + \tau_1 X_{34} + \tau_2 Y_{34}, \\ \Phi_{44} = -R + \tau_1 X_{44} + \tau_2 Y_{44}, \\ S = R + \tau_1 X_{55} + \tau_2 Y_{55}. \end{cases} \tag{8}$$

Proof: Choose the candidate Lyapunov functional to be

$$\begin{aligned} V(x_t) \quad := \quad & (\mathcal{D}x_t)^T P \mathcal{D}x_t + \int_{t-\tau_1}^{t} x^T(s) Q_1 x(s) ds + \int_{t-\tau_2}^{t} x^T(s) Q_2 x(s) ds + \int_{t-\tau_2}^{t} \dot{x}^T(s) R \dot{x}(s) ds \\ & + \int_{-\tau_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) X_{55} \dot{x}(s) ds d\theta + \int_{-\tau_2}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Y_{55} \dot{x}(s) ds d\theta, \end{aligned} \tag{9}$$

where $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1,\ 2)$, $R = R^T > 0$ are weighting matrices, and $X_{55} = X_{55}^T \geq 0$, $Y_{55} = Y_{55}^T \geq 0$. All of these matrices need to be determined. Calculating the derivative of $V(x_t)$ along the solutions of Σ_0 yields

$$\begin{aligned} \dot{V}(x_t) \quad = \quad & 2(\mathcal{D}x_t)^T P \left[Ax(t) + Bx(t-\tau_1)\right] + x^T(t) Q_1 x(t) - x^T(t-\tau_1) Q_1 x(t-\tau_1) \\ & + x^T(t) Q_2 x(t) - x^T(t-\tau_2) Q_2 x(t-\tau_2) + \dot{x}^T(t) R \dot{x}(t) - \dot{x}^T(t-\tau_2) R \dot{x}(t-\tau_2) \\ & + \tau_1 \dot{x}^T(t) X_{55} \dot{x}(t) - \int_{t-\tau_1}^{t} \dot{x}^T(s) X_{55} \dot{x}(s) ds \\ & + \tau_2 \dot{x}^T(t) Y_{55} \dot{x}(t) - \int_{t-\tau_2}^{t} \dot{x}^T(s) Y_{55} \dot{x}(s) ds. \end{aligned} \tag{10}$$

Using the Leibniz-Newton formula, we can write

$$x(t-\tau_1) = x(t) - \int_{t-\tau_1}^{t} \dot{x}(s) ds, \tag{11}$$

$$x(t-\tau_2) = x(t) - \int_{t-\tau_2}^{t} \dot{x}(s) ds. \tag{12}$$

According to Eqs. (11) and (12), for any matrices $X_{i5}, Y_{i5}\,(i=1,\cdots,4)$, the following equations hold.

$$2\left[x^T(t)X_{15}+x^T(t-\tau_1)X_{25}+x(t-\tau_2)X_{35}+\dot{x}(t-\tau_2)X_{45}\right]\left[x(t)-x(t-\tau_1)-\int_{t-\tau_1}^{t}\dot{x}(s)ds\right]=0, \tag{13}$$

$$2\left[x^T(t)Y_{15}+x^T(t-\tau_1)Y_{25}+x^T(t-\tau_2)Y_{35}+\dot{x}(t-\tau_2)Y_{45}\right]\left[x(t)-x(t-\tau_2)-\int_{t-\tau_2}^{t}\dot{x}(s)ds\right]=0. \tag{14}$$

On the other hand, for any appropriately dimensioned matrices X_{ij}, Y_{ij} $(i=1,\cdots,4;\, i\le j\le 4)$, the following equation also holds.

$$\begin{bmatrix} x(t) \\ x(t-\tau_1) \\ x(t-\tau_2) \\ \dot{x}(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} \\ \Lambda_{12}^T & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} \\ \Lambda_{13}^T & \Lambda_{23}^T & \Lambda_{33} & \Lambda_{34} \\ \Lambda_{14}^T & \Lambda_{24}^T & \Lambda_{34}^T & \Lambda_{44} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_1) \\ x(t-\tau_2) \\ \dot{x}(t-\tau_2) \end{bmatrix} = 0, \tag{15}$$

where

$$\Lambda_{ij}=\tau_1(X_{ij}-X_{ij})+\tau_2(Y_{ij}-Y_{ij}),\ i=1,\cdots,4;\ i\le j\le 4.$$

Then, we add the terms on the left sides of Eqs. (13)-(15) to $\dot{V}(x_t)$; and consider the fact that, for any $r\ge 0$ and any function $f(t)$,

$$\int_{t-r}^{t} f(t)ds = rf(t),$$

$\dot{V}(x_t)$ can be expressed as follows:

$$\dot{V}(x_t)=z_1^T(t)\Omega z_1(t)-\int_{t-\tau_1}^{t} z_2^T(t,s)\Psi z_2(t,s)ds-\int_{t-\tau_2}^{t} z_2^T(t,s)\Xi z_2(t,s)ds, \tag{16}$$

where

$$z_1(t)=\left[\,x^T(t)\quad x^T(t-\tau_1)\quad x^T(t-\tau_2)\quad \dot{x}^T(t-\tau_2)\,\right]^T,\ z_2(t,s)=\left[\,z_1^T(t)\quad \dot{x}^T(s)\,\right]^T,$$

$$\Omega=\begin{bmatrix} \Phi_{11}+A^TSA & \Phi_{12}+A^TSB & \Phi_{13} & \Phi_{14}+A^TSC \\ \Phi_{12}^T+B^TSA & \Phi_{22}+B^TSB & \Phi_{23} & \Phi_{24}+B^TSC \\ \Phi_{13}^T & \Phi_{23}^T & \Phi_{33} & \Phi_{34} \\ \Phi_{14}^T+C^TSA & \Phi_{24}^T+C^TSB & \Phi_{34}^T & \Phi_{44}+C^TSC \end{bmatrix},$$

and Ψ, Ξ, $\Phi_{ij}\,(i=1,\cdots,4;\, i\le j\le 4)$ and S are defined in (5)-(8). If $\Omega<0$, $\Psi\ge 0$ and $\Xi\ge 0$, then $\dot{V}(x_t)<0$ for any $z_1(t)\ne 0$. Applying Schur complements, $\Phi<0$ implies that $\Omega<0$. So Σ_0 is asymptotically stable if the LMIs (5)-(7) are feasible. ∎

Remark 1: In Theorem 1, the relationships between the terms $x(t-\tau_1)$ and $x(t)-\int_{t-\tau_1}^{t}\dot{x}(s)ds$, and $x(t-\tau_2)$ and $x(t)-\int_{t-\tau_2}^{t}\dot{x}(s)ds$ have been considered through the free weighting matrices, X_{i5} and Y_{i5} $(i=1,\cdots,4)$; and the optimal weighting matrices can be selected by solving the LMIs (6) and (7). In contrast, previous methods employed fixed weighting matrices, which are not usually the optimal ones.

Remark 2: In previous methods, it is very hard to handle the case where the neutral and discrete delays are different. As can be seen in Eq. (10), if we use the Leibniz-Newton formula to replace $x(t-\tau_1)$ and $x(t-\tau_2)$ in $\mathcal{D}x_t$ in the first term on the right side, a multiplication term involving two integrals appears, which is very difficult to deal with. To avoid this situation,

$x(t-\tau_1)$ was replaced in some places using the Leibniz-Newton formula, but was retained $x(t-\tau_2)$ in $\mathcal{D}x_t$. Consequently, when that method is applied to a system with different neutral and discrete delays, the results are all discrete-delay-dependent and neutral-delay-independent. In Theorem 1, both $x(t-\tau_1)$ and $x(t-\tau_2)$ in $\mathcal{D}x_t$ are retained, but the relationships between the terms in the Leibniz-Newton formula are expressed in terms of free weighting matrices. This method overcomes the difficulty that appears in previous studies when the neutral and discrete delays are different. Thus, the criterion in Theorem 1 includes information on the sizes of τ_1 and τ_2, which makes it both a neutral- and a discrete-delay-dependent stability criterion. So, this method yields a less conservative criterion than previous discrete-delay-dependent and neutral-delay-independent criteria.

From Theorem 1, we obtain a delay-dependent criterion for the neutral system Σ with time-varying structured uncertainties.

Theorem 2 *Given scalars $\tau_1 > 0$ and $\tau_2 > 0$, the neutral system Σ with time-varying structured uncertainties is robustly stable if the operator $\mathcal{D}$ is stable and there exist positive definite matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1, 2)$ and $R = R^T > 0$, non-negative definite matrices $X_{ii} = X_{ii}^T \geq 0$ and $Y_{ii} = Y_{ii}^T \geq 0 (i = 1, \cdots, 5)$, any matrices X_{ij} and $Y_{ij} (i = 1, \cdots, 5; i < j \leq 5)$ and a scalar $\lambda > 0$ such that the LMIs (6), (7) and (17) are feasible.*

$$\begin{bmatrix} \Phi_{11} + \lambda E_a^T E_a & \Phi_{12} + \lambda E_a^T E_b & \Phi_{13} & \Phi_{14} & A^T S & PD \\ \Phi_{12}^T + \lambda E_b^T E_a & \Phi_{22} + \lambda E_b^T E_b & \Phi_{23} & \Phi_{24} & B^T S & 0 \\ \Phi_{13}^T & \Phi_{23}^T & \Phi_{33} & \Phi_{34} & 0 & -C^T PD \\ \Phi_{14}^T & \Phi_{24}^T & \Phi_{34}^T & \Phi_{44} & C^T S & 0 \\ SA & SB & 0 & SC & -S & SD \\ D^T P & 0 & -D^T PC & 0 & D^T S & -\lambda I \end{bmatrix} < 0, \qquad (17)$$

where Φ_{ij} $(i = 1, \cdots, 4; i \leq j \leq 4)$ and S are given in (8).

Proof: If A and B in (5) are replaced with $A + DF(t)E_a$ and $B + DF(t)E_b$, respectively, then (5) for the uncertain system Σ is equivalent to the following condition.

$$\Phi + \Gamma_d^T F(t) \Gamma_e + \Gamma_e^T F^T(t) \Gamma_d < 0. \qquad (18)$$

where

$$\Gamma_d = \begin{bmatrix} D^T P & 0 & -D^T PC & 0 & D^T S \end{bmatrix},$$

$$\Gamma_e = \begin{bmatrix} E_a & E_b & 0 & 0 & 0 \end{bmatrix}..$$

By Lemma 1, a necessary and sufficient condition for (18) for Σ is that there exists a $\lambda > 0$ such that

$$\Phi + \lambda^{-1} \Gamma_d^T \Gamma_d + \lambda \Gamma_e^T \Gamma_e < 0. \qquad (19)$$

Applying Schur complements, we find that (19) is equivalent to (17). ∎

Remark 3: The above results can easily be extended to a neutral system with multiple discrete- and neutral-delays.

4 Examples

This section presents some examples to illustrate the effectiveness of the method described above.

Example 1([3], [5], [17]): Consider the stability of the nominal system Σ_0 with

$$A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix},\ A_1 = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},\ C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}.$$

The upper bounds on the delays that guarantee the stability of this system in [17], [3] and [5] are $\tau_1 = \tau_2 = 0.3$, $\tau_1 = \tau_2 = 0.5658$ and $\tau_1 = \tau_2 = 0.74$, respectively. In contrast, solving LMIs (5)-(7) for $\tau_1 = \tau_2$, we obtained the maximum upper bounds on the allowable sizes to be $\tau_1 = \tau_2 = 1.6527$, which are 450.9%, 192.1% and 123.3% larger than those in [17], [3] and [5], respectively. This means that our method is less conservative than previous methods. On the other hand, we also obtained the values for $\tau_1 \neq \tau_2$. Table 1 lists the upper bounds on τ_1 that guarantee the stability of the system for values of τ_2 from 0.1 to 1.6. It can be seen that the upper bound on τ_1 decreases as τ_2 increases when τ_2 is small, but that τ_1 remains almost unchanged when $\tau_2 \geq 1.2$.

Table 1: Calculated allowable size of discrete delay τ_1 (Example 1).

τ_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
τ_1	1.7100	1.6987	1.6883	1.6792	1.6718	1.6664	1.6624	1.6591	1.6564
τ_2	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.6527	10000
τ_1	1.6543	1.6531	1.6527	1.6527	1.6527	1.6527	1.6527	1.6527	1.6527

Example 2: Consider the robust stability of the system Σ with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix},\ 0 \leq c < 1,$$

$$D = I,\ E_a = E_b = 0.2I.$$

Table 2 shows some calculation results obtained from Theorem 2, and compares them to those obtained by the method of [9]. The values in the table are the maximum upper bounds on the delay τ_1. It is clear that our results are significantly better when $\tau_1 = \tau_2$. On the other hand, when $\tau_1 \neq \tau_2$, if the maximum upper bound on the delay τ_2 is small, then the maximum upper bound on τ_1 is larger than that when $\tau_1 = \tau_2$. It can also be seen that the change in τ_2 has a big effect on the upper bound on τ_1 when τ_2 is small, but only a small effect when it is large.

Table 2: Calculated allowable size of discrete delay τ_1 (Example 2).

c	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Han's paper [9] ($\tau_1 = \tau_2$)	1.77	1.63	1.48	1.33	1.16	0.98	0.79	0.59	0.37
Theorem 2 ($\tau_1 = \tau_2$)	2.39	2.05	1.75	1.49	1.27	1.08	0.91	0.76	0.63
Theorem 2 ($\tau_2 = 10000$)	2.39	2.05	1.75	1.49	1.27	1.08	0.91	0.76	0.63
Theorem 2 ($\tau_2 = 0.1$)	2.39	2.25	2.11	1.96	1.81	1.66	1.50	1.33	1.16

5 Conclusion

This paper presents some new criteria for the delay-dependent robust stability of neutral systems with mixed delays and time-varying structured uncertainties. The criteria take into account the relationships between $x(t-\tau_1)$ and $x(t) - \int_{t-\tau_1}^{t} \dot{x}(s)ds$, $x(t-\tau_2)$ and $x(t) - \int_{t-\tau_2}^{t} \dot{x}(s)ds$. The free weighting matrices used to express the relationships between, and the reciprocal influences of, these terms are selected by means of LMIs. The criteria arrived at in this paper are both neutral- and discrete-delay-dependent. Numerical examples illustrate both the improvement that this method provides over previous methods, and also the reciprocal influences between neutral delays and discrete delays.

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