New Delay-Dependent Stability Criteria and Stabilizing Method for Neutral Systems

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***Abstract*— This paper concerns delay-dependent robust stability criteria and a design method for stabilizing neutral systems with time-varying structured uncertainties. A new way of deriving such criteria is presented that combines the parameterized model transformation method with a method that takes the relationships between the terms in the Leibniz-Newton formula into account. The relationships are expressed as free weighting matrices obtained by solving LMIs. Moreover, the stability criteria are also used to design a stabilizing state-feedback controller. Numerical examples illustrate the effectiveness of the method and the improvement over some existing methods.**

***Index Terms*— neutral system, time-varying structured uncertainties, robust stability, delay-dependent criterion, state feedback stabilizing controller, linear matrix inequality (LMI).**

I. Introduction

STABILITY criteria for neutral systems can be classified into two types: delay-dependent, which include information on the size of delays, [1]–[10], and delay-independent, which are applicable to delays of arbitrary size [11]. Delay-independent stability criteria tend to be conservative, especially for small delays, while delay-dependent ones are usually less conservative.

The Lyapunov functional method is the main method employed to derive delay-dependent criteria. The discretized-Lyapunov-functional method (e.g., [5], [12], [13]) is one of the most efficient among them, but it is difficult to extend to the synthesis of a control system. Another method involves a fixed model transformation, which expresses the delay term in terms of an integral. Four basic model transformations have been proposed [9]. The descriptor model transformation method combined with Park's or Moon *et al.*'s inequalities [14], [15] is the most efficient [8], [9], [16]. But there is room for further investigation. For example, in the derivative of the Lyapunov functional, the Leibniz-Newton formula was used, and the term $x(t-\tau)$ was replaced by $x(t)-\int_{t-\tau}^{t}\dot{x}(s)ds$ in some places but not in others. Moreover, the relationship between these two terms was not considered. Recently, He *et al.* [10] devised a new method that employs free weighting matrices to express the relationships between the terms in the Leibniz- Newton formula. This overcomes the conservativeness of methods involving a fixed model transformation.

A different idea is the application of a parameterized model transformation with a parameter matrix. The delayed matrix (the coefficient matrix of the delayed term) is decomposed into two parts. One part is kept; and the other part is replaced either with $x(t)-\int_{t-\tau}^{t}\dot{x}(s)ds$, which is in the derivative of the Lyapunov functional [6], or with the neutral transformation [4]. However, in the former treatment [6], the weighting matrices are fixed, as in [8], [9], [14]–[16]; and in both treatments, the method of decomposing the parameter matrix [4], [6] needs more investigation. Han presented a method of selecting the parameter matrix (Remark 7) in [6]; but a severe restriction was imposed, namely, that three of the matrices must be chosen to be the same, which may lead to conservativeness.

This paper presents a new parameterized-matrix form expressed in terms of the solution of a linear matrix inequality (LMI) [17]. This is combined with the free-weighting-matrix method [10] to yield a new stability criterion for a neutral system with no uncertainties. The criterion is further extended to a system with time-varying structured uncertainties. Based on this criterion, a method of designing a stabilizing state feedback controller is derived.

II. Notation and Preliminaries

Consider the following neutral system, Σ, with time-varying structured uncertainties.

$$\Sigma:\begin{cases}\dot{x}(t)-C\dot{x}(t-\tau)=(A+\Delta A(t))x(t)\\ \quad +(A_d+\Delta A_d(t))x(t-\tau)+Bu(t),\ t>0,\\ x(t)=0,\ t\in[-\tau,0],\end{cases}\tag{1}$$

where $x(t)\in\mathcal{R}^n$ is the state vector; $u(t)\in\mathcal{R}^m$ is the control input; $\tau\geq 0$ is a constant time delay; and A, A_d, C and B are constant matrices with appropriate dimensions. The uncertainties are of the form

$$[\Delta A(t)\ \Delta A_d(t)]=HF(t)[E_a\ E_{ad}],\tag{2}$$

where H, E_a and E_{ad} are appropriately dimensioned constant matrices, and $F(t)$ is an unknown real and possibly time-varying matrix with Lebesgue-measurable elements satisfying

$$\|F(t)\|\leq 1,\ \forall t,\tag{3}$$

where $\|\cdot\|$ is the Euclidean norm.

The problem is to find a state feedback gain, $K\in\mathcal{R}^{m\times n}$, in the control law

$$u(t)=Kx(t)\tag{4}$$

that stabilizes Σ.

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First, the nominal system, Σ_0, of Σ is discussed. It is given by

$$\Sigma_0 : \begin{cases} \dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + A_d x(t-\tau) \\ \qquad\qquad +Bu(t),\ t > 0, \\ x(t) = 0,\ t \in [-\tau, 0]. \end{cases} \quad (5)$$

The following lemma is used to deal with a system with time-varying uncertainties [10].

Lemma 1: Given matrices $Q = Q^T, H, E$, and $R = R^T > 0$ with appropriate dimensions,

$$Q + HF(t)E + E^T F^T(t) H^T < 0$$

for all $F(t)$ satisfying $F^T(t)F(t) \leq R$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$Q + \varepsilon^{-1} HH^T + \varepsilon E^T R E < 0.$$

The operator $\mathcal{D}$: $C([-\tau, 0], R^n) \to R^n$ is defined to be

$$\mathcal{D}x_t = x(t) - Cx(t-\tau).$$

Its stability is defined as follows [18]:

Definition 1: The operator $\mathcal{D}$ is said to be stable if the zero solution of the homogeneous difference equation $\mathcal{D}x_t = 0,\ t \geq 0,\ x_0 = \psi \in \{\phi \in C([-\tau, 0] : \mathcal{D}\phi = 0\}$ is uniformly asymptotically stable.

III. Stability Issues

This section discusses the stability of Σ_0 and Σ with $u(t) = 0$.

A. Asymptotic Stability

First, a delay-dependent stability criterion for Σ_0 is presented.

Theorem 1: Given a scalar $\tau \geq 0$, the nominal neutral system, Σ_0, of Σ with $u(t) = 0$ is asymptotically stable if the operator $\mathcal{D}$ is stable and there exist $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $Z = Z^T \geq 0$, $W = W^T \geq 0$, and any matrices N_i and T_i $(i = 1, \cdots, 4)$ with appropriate dimensions such that the following LMI holds.

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \tau P_{22} & -\tau N_1 \\ \Phi_{12}^T & \Phi_{22} & \Phi_{23} & \Phi_{24} & \tau P_{12} & -\tau N_2 \\ \Phi_{13}^T & \Phi_{23}^T & \Phi_{33} & \Phi_{34} & -\tau P_{22} & -\tau N_3 \\ \Phi_{14}^T & \Phi_{24}^T & \Phi_{34}^T & \Phi_{44} & -\tau C^T P_{12} & -\tau N_4 \\ \tau P_{22} & \tau P_{12}^T & -\tau P_{22} & -\tau P_{12}^T C & -\tau W & 0 \\ -\tau N_1^T & -\tau N_2^T & -\tau N_3^T & -\tau N_4^T & 0 & -\tau Z \end{bmatrix} < 0, \quad (6)$$

where

$$\Phi_{11} = P_{12} + P_{12}^T + Q + \tau W + N_1 + N_1^T - T_1 A - A^T T_1^T,$$
$$\Phi_{12} = P_{11} + N_2^T + T_1 - A^T T_2^T,$$
$$\Phi_{13} = -P_{12} - P_{12}^T C + N_3^T - N_1 - T_1 A_d - A^T T_3^T,$$
$$\Phi_{14} = -P_{11} C + N_4^T - T_1 C - A^T T_4^T,$$
$$\Phi_{22} = R + \tau Z + T_2 + T_2^T,$$
$$\Phi_{23} = -P_{11} C - N_2 - T_2 A_d + T_3^T,$$
$$\Phi_{24} = -T_2 C + T_4^T,$$
$$\Phi_{33} = -Q + P_{12}^T C + C^T P_{12} - N_3 - N_3^T - A_d^T T_3^T - T_3 A_d,$$
$$\Phi_{34} = C^T P_{11} C - N_4^T - T_3 C - A_d^T T_4^T,$$
$$\Phi_{44} = -R - T_4 C - C^T T_4^T.$$

Proof: Choose a Lyapunov functional candidate to be

$$\begin{aligned} V(x_t) \;:=\; & (\mathcal{D}x_t)^T P_{11} (\mathcal{D}x_t) + 2(\mathcal{D}x_t)^T P_{12} \int_{t-\tau}^{t} x(s)ds \\ & + \left[\int_{t-\tau}^{t} x(s)ds\right]^T P_{22} \int_{t-\tau}^{t} x(s)ds \\ & + \int_{t-\tau}^{t} x^T(s)Qx(s)ds + \int_{t-\tau}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\ & + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z\dot{x}(s)dsd\theta \\ & + \int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s)Wx(s)dsd\theta, \end{aligned} \quad (7)$$

where $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0, Q = Q^T > 0, R = R^T > 0, Z = Z^T \geq 0$, and $W = W^T \geq 0$ are to be determined. It is easy to verify that $V(x_t)$ satisfies the condition

$$\alpha_1 \|\mathcal{D}x_t\|^2 \leq V(x_t) \leq \alpha_2 \|x_t\|_{c1}^2,$$

where $\|x_t\|_{c1} := sup_{-\tau \leq \theta \leq 0}\{\|x(t+\theta)\|, \|\dot{x}(t+\theta)\|\}$ and $\alpha_1 = \lambda_{min}(P)$, $\alpha_2 = \lambda_{max}(P)\{1 + \|C\| + \tau\} + \tau\{\lambda_{max}(Q) + \lambda_{max}(R)\} + \frac{1}{2}\tau^2\{\lambda_{max}(Z) + \lambda_{max}(W)\}$ [1].

From the Leibniz-Newton formula, the following equation is true for any matrices N_i $(i = 1, \cdots, 4)$.

$$2\left[x^T(t)N_1 + \dot{x}^T(t)N_2 + x^T(t-\tau)N_3 + \dot{x}^T(t-\tau)N_4\right] \times \left[x(t) - \int_{t-\tau}^{t} \dot{x}(s)ds - x(t-\tau)\right] = 0. \quad (8)$$

And from the system definition (5), the following equation is also true for any matrices T_i $(i = 1, \cdots, 4)$.

$$2\left[x^T(t)T_1 + \dot{x}^T(t)T_2 + x^T(t-\tau)T_3 + \dot{x}^T(t-\tau)T_4\right] \times \cdot [\dot{x}(t) - C\dot{x}(t-\tau) - Ax(t) - A_d x(t-\tau)] = 0. \quad (9)$$

Calculating the derivative of $V(x_t)$ along the solution of Σ_0 yields

$$\begin{aligned} \dot{V}(x_t) = & 2[x(t) - Cx(t-\tau)]^T P_{11} [\dot{x}(t) - C\dot{x}(t-\tau)] \\ & +2[\dot{x}(t) - C\dot{x}(t-\tau)]^T P_{12} \int_{t-\tau}^{t} x(s)ds \\ & +2[x(t) - Cx(t-\tau)]^T P_{12}[x(t) - x(t-\tau)] \\ & +2[x(t) - x(t-\tau)]^T P_{22} \int_{t-\tau}^{t} x(s)ds + x^T(t)Qx(t) \\ & -x^T(t-\tau)Qx(t-\tau) + \dot{x}^T(t)R\dot{x}(t) \\ & -\dot{x}^T(t-\tau)R\dot{x}(t-\tau) + \tau\dot{x}^T(t)Z\dot{x}(t) \\ & -\int_{t-\tau}^{t} \dot{x}^T(s)Z\dot{x}(s)ds + \tau x^T(t)Wx(t) \end{aligned}$$

[1] Some connections between the stability results for the norms $\|\cdot\|_c$ and $\|\cdot\|_{c1}$ can be found in [19].

$$
\begin{aligned}
&-\int_{t-\tau}^{t} x^T(s)Wx(s)ds \\
&+2\left[x^T(t)N_1+\dot{x}^T(t)N_2+x^T(t-\tau)N_3+\dot{x}^T(t-\tau)N_4\right]\times \\
&\left[x(t)-\int_{t-\tau}^{t}\dot{x}(s)ds-x(t-\tau)\right] \\
&+2\left[x^T(t)T_1+\dot{x}^T(t)T_2+x^T(t-\tau)T_3+\dot{x}^T(t-\tau)T_4\right]\times \\
&\left[\dot{x}(t)-C\dot{x}(t-\tau)-Ax(t)-A_dx(t-\tau)\right] \\
:=&\ \frac{1}{\tau}\int_{t-\tau}^{t}\zeta^T(t,s)\Phi\zeta(t,s)ds,
\end{aligned}
\tag{10}
$$

where $\zeta(t,s) = [x^T(t)\ \dot{x}^T(t)\ x^T(t-\tau)\ \dot{x}^T(t-\tau)\ x^T(s)\ \dot{x}^T(s)]^T$, and Φ is defined in (6). If $\Phi < 0$, then $\dot{V}(x_t) \leq -\varepsilon\|x(t)\|^2$ for a sufficiently small $\varepsilon > 0$. Since $\mathcal{D}$ is stable, Σ is asymptotically stable if LMI (6) holds. ■

In fact, P in Theorem 1 can be chosen to be semi-positive. For example, selecting $P_{12} = 0, P_{22} = 0$ and $W = 0$ yields the following criterion. Note that these values result in a different Lyapunov functional.

Corollary 1: Given a scalar $\tau \geq 0$, the nominal neutral system Σ_0 with $u(t) = 0$ is asymptotically stable if the operator $\mathcal{D}$ is stable and there exist $P_{11} = P_{11}^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $Z = Z^T \geq 0$, and any matrices N_i and T_i $(i = 1, \cdots, 4)$ with appropriate dimensions such that the following LMI holds.

$$
\Xi = \begin{bmatrix}
\Xi_{11} & \Phi_{12} & \Xi_{13} & \Phi_{14} & -\tau N_1 \\
\Phi_{12}^T & \Phi_{22} & \Phi_{23} & \Phi_{24} & -\tau N_2 \\
\Xi_{13}^T & \Phi_{23}^T & \Xi_{33} & \Phi_{34} & -\tau N_3 \\
\Phi_{14}^T & \Phi_{24}^T & \Phi_{34}^T & \Phi_{44} & -\tau N_4 \\
-\tau N_1^T & -\tau N_2^T & -\tau N_3^T & -\tau N_4^T & -\tau Z
\end{bmatrix} < 0,
\tag{11}
$$

where

$$
\begin{aligned}
\Xi_{11} &= Q + N_1 + N_1^T - T_1A - A^TT_1^T, \\
\Xi_{13} &= N_3^T - N_1 - T_1A_d - A^TT_3^T, \\
\Xi_{33} &= -Q - N_3 - N_3^T - A_d^TT_3^T - T_3A_d,
\end{aligned}
$$

and Φ_{ij} $(i = 1, \cdots, 4; i \leq j \leq 4)$ are defined in Theorem 1.

Remark 1: In the above theorem and corollary, the free weighting matrices N_i $(i = 1, \cdots, 4)$ in (8) express the relationships between the items $x(t)$, $x(t-\tau)$, and $\int_{t-\tau}^{t}\dot{x}(s)ds$, and are obtained by solving the LMI. In fact, Corollary 1 can be derived directly by the free-weighting-matrix method [10]. Thus, the matrices P_{12}, P_{22} and W in Theorem 1, which are obtained by solving the LMI, provide extra freedom.

On the other hand, if we choose $Z = 0$ and $N_i = 0$ $(i = 1, \cdots, 4)$, another criterion can also be derived.

Corollary 2: Given a scalar $\tau \geq 0$, the nominal neutral system Σ_0 with $u(t) = 0$ is asymptotically stable if the operator $\mathcal{D}$ is stable and there exist $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $W = W^T \geq 0$, and any matrices T_i $(i = 1, \cdots, 4)$ with appropriate dimensions such that the following LMI holds.

$$
\Psi = \begin{bmatrix}
\Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \tau P_{22} \\
\Psi_{12}^T & \Psi_{22} & \Psi_{23} & \Phi_{24} & \tau P_{12} \\
\Psi_{13}^T & \Psi_{23}^T & \Psi_{33} & \Psi_{34} & -\tau P_{22} \\
\Psi_{14}^T & \Phi_{24}^T & \Psi_{34}^T & \Phi_{44} & -\tau C^TP_{12} \\
\tau P_{22} & \tau P_{12}^T & -\tau P_{22} & -\tau P_{12}^TC & -\tau W
\end{bmatrix} < 0,
\tag{12}
$$

where

$$
\begin{aligned}
\Psi_{11} &= P_{12} + P_{12}^T + Q + \tau W - T_1A - A^TT_1^T, \\
\Psi_{12} &= P_{11} + T_1 - A^TT_2^T, \\
\Psi_{13} &= -P_{12} - P_{12}^TC - T_1A_d - A^TT_3^T, \\
\Psi_{14} &= -P_{11}C - T_1C - A^TT_4^T, \Psi_{22} = R + T_2 + T_2^T, \\
\Psi_{23} &= -P_{11}C - T_2A_d + T_3^T, \\
\Psi_{33} &= -Q + P_{12}^TC + C^TP_{12} - A_d^TT_3^T - T_3A_d, \\
\Psi_{34} &= C^TP_{11}C - T_3C - A_d^TT_4^T,
\end{aligned}
$$

and Φ_{24} and Φ_{44} are defined in Theorem 1.

Remark 2: Corollary 2 is, in fact, a parameterized model transformation. The parameter matrices are combined into the Lyapunov matrices, P_{12} and P_{22}, in the Lyapunov functional, and are obtained by solving the LMI. From Corollaries 1 and 2, it is clear that Theorem 1 is a combination of the free- weighting-matrix method and a parameterized model transformation.

B. Robust Stability

Extending Theorem 1 to a neutral system with time-varying structured uncertainties yields the following delay-dependent robust stability criterion.

Theorem 2: Given a scalar $\tau \geq 0$, the neutral system Σ with $u(t) = 0$ is robustly stable if the operator $\mathcal{D}$ is stable and there exist $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $Z = Z^T \geq 0$, $W = W^T \geq 0$, and any matrices N_i and T_i $(i = 1, \cdots, 4)$ with appropriate dimensions such that the following LMI holds.

$$
\Pi = \begin{bmatrix}
\Pi_{11} & \Phi_{12} & \Pi_{13} & \Phi_{14} & \tau P_{22} & -\tau N_1 & -T_1H \\
\Phi_{12}^T & \Phi_{22} & \Phi_{23} & \Phi_{24} & \tau P_{12} & -\tau N_2 & -T_2H \\
\Pi_{13}^T & \Phi_{23}^T & \Pi_{33} & \Phi_{34} & -\tau P_{22} & -\tau N_3 & -T_3H \\
\Phi_{14}^T & \Phi_{24}^T & \Phi_{34}^T & \Phi_{44} & -\tau C^TP_{12} & -\tau N_4 & -T_4H \\
\tau P_{22} & \tau P_{12}^T & -\tau P_{22} & -\tau P_{12}^TC & -\tau W & 0 & 0 \\
-\tau N_1^T & -\tau N_2^T & -\tau N_3^T & -\tau N_4^T & 0 & -\tau Z & 0 \\
-H^TT_1^T & -H^TT_2^T & -H^TT_3^T & -H^TT_4^T & 0 & 0 & -I
\end{bmatrix} < 0,
\tag{13}
$$

where

$$
\begin{aligned}
&\Pi_{11} = \Phi_{11} + E_a^TE_a, \quad \Pi_{13} = \Phi_{13} + E_a^TE_{ad}, \\
&\Pi_{33} = \Phi_{33} + E_{ad}^TE_{ad},
\end{aligned}
$$

and Φ_{ij} $(i = 1, \cdots, 4; i \leq j \leq 4)$ are defined in (6).

Proof: Replacing A and A_d in (6) with $A + HF(t)E_a$ and $A_d + HF(t)E_{ad}$, respectively, we find that (6) for Σ is equivalent to the following condition.

$$\Phi + \Gamma_h^T F(t)\Gamma_e + \Gamma_e^T F^T(t)\Gamma_h < 0, \tag{14}$$

where

$$\begin{aligned}\Gamma_h &= \begin{bmatrix} -H^T T_1^T & -H^T T_2^T & -H^T T_3^T & -H^T T_4^T & 0 & 0 \end{bmatrix},\\ \Gamma_e &= \begin{bmatrix} E_a & 0 & E_{ad} & 0 & 0 & 0 \end{bmatrix}.\end{aligned}$$

By Lemma 1, a sufficient condition guaranteeing (14) is that there exists a scalar $\lambda > 0$ such that

$$\Phi + \lambda\Gamma_h^T\Gamma_h + \lambda^{-1}\Gamma_e^T\Gamma_e < 0. \tag{15}$$

That is,

$$\lambda\Phi + \lambda^2\Gamma_h^T\Gamma_h + \Gamma_e^T\Gamma_e < 0. \tag{16}$$

Replacing $\lambda P, \lambda Q, \lambda R, \lambda Z, \lambda W$, λN_i and λT_i $(i = 1, \cdots, 4)$ with P, Q, R, Z, W, N_i and T_i $(i = 1, \cdots, 4)$, respectively, and applying the Schur complement [17] shows that (**??**) is equivalent to (13). ■

Remark 3: The criteria obtained in Corollaries 1 and 2 can also be extended to a system with time-varying structured uncertainties in the same manner.

IV. State feedback control

The results in the previous section can also be used to verify the stability of the closed-loop systems Σ_0 and Σ with (4), and to design a stabilizing state feedback controller (4).

The following theorem holds for Σ_0.

Theorem 3: Given scalars $\tau \geq 0$ and t_i $(i = 1, \cdots, 4)$, the control law (4) stabilizes the nominal neutral system Σ_0 if the operator $\mathcal{D}$ is stable and there exist $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $Z = Z^T \geq 0$, $W = W^T \geq 0$, and any matrices N_i $(i = 1, \cdots, 4)$, S and V with appropriate dimensions such that the following LMI holds.

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \tau P_{22} & -\tau N_1 \\ \Theta_{12}^T & \Theta_{22} & \Theta_{23} & \Theta_{24} & \tau P_{12} & -\tau N_2 \\ \Theta_{13}^T & \Theta_{23}^T & \Theta_{33} & \Theta_{34} & -\tau P_{22} & -\tau N_3 \\ \Theta_{14}^T & \Theta_{24}^T & \Theta_{34}^T & \Theta_{44} & -\tau CP_{12} & -\tau N_4 \\ \tau P_{22} & \tau P_{12}^T & -\tau P_{22} & -\tau P_{12}^T C^T & -\tau W & 0 \\ -\tau N_1^T & -\tau N_2^T & -\tau N_3^T & -\tau N_4^T & 0 & -\tau Z \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{aligned}
\Theta_{11} &= P_{12} + P_{12}^T + Q + \tau W + N_1 + N_1^T - t_1(AS^T + BV) \\ &\quad - t_1(SA^T + V^T B^T),\\
\Theta_{12} &= P_{11} + N_2^T + t_1 S - t_2(AS^T + BV),\\
\Theta_{13} &= -P_{12} - P_{12}^T C^T + N_3^T - N_1 - t_1 S A_d^T - t_3(AS^T + BV),\\
\Theta_{14} &= -P_{11}C^T + N_4^T - t_1 SC^T - t_4(AS^T + BV),\\
\Theta_{22} &= R + \tau Z + t_2(S + S^T),\\
\Theta_{23} &= -P_{11}C^T - N_2 - t_2 S A_d^T + t_3 S^T,\\
\Theta_{24} &= -t_2 SC^T + t_4 S^T,\\
\Theta_{33} &= -Q + P_{12}^T C^T + CP_{12} - N_3 - N_3^T - t_3(A_d S^T + S A_d^T),\\
\Theta_{34} &= CP_{11}C^T - N_4^T - t_3 SC^T - t_4 A_d S^T,\\
\Theta_{44} &= -R - t_4(SC^T + CS^T).
\end{aligned}$$

Moreover, a stabilizing control law is given by $u(t) = VS^{-T}x(t)$.

Proof: Applying the control law (4) to Σ_0 yields

$$\dot{x}(t) - C\dot{x}(t-\tau) = (A + BK)x(t) + A_d x(t-\tau). \tag{18}$$

Since the solution of $\det|sI - (A+BK) - A_d e^{-\tau s} - sCe^{-\tau s}| = 0$ is the same as that of $\det|sI - (A + BK)^T - A_d^T e^{-\tau s} - sC^T e^{-\tau s}| = 0$, as long as stability is the only concern, (18) is equivalent to the system

$$\dot{y}(t) - C^T\dot{y}(t-\tau) = (A + BK)^T y(t) + A_d^T y(t-\tau). \tag{19}$$

Hence, replacing A, A_d and C in (6) with $(A + BK)^T, A_d^T$ and C^T, respectively, and setting $T_1 = t_1 S, T_2 = t_2 S, T_3 = t_3 S, T_4 = t_4 S$ and $V = KS^T$ yields (17). Since Θ_{22} in (17) must be negative definite, the same is true for $t_2(S + S^T)$. So, S is nonsingular. Thus, $u(t) = VS^{-T}x(t)$. ■

Next, a stabilizing memoryless controller (4) for Σ is designed as follows:

Theorem 4: Given scalars $\tau \geq 0$ and t_i $(i = 1, \cdots, 4)$, the control law (4) stabilizes the nominal neutral system Σ_0 if the operator $\mathcal{D}$ is stable and there exist $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0$, $Q = Q^T > 0$, $R = R^T > 0$, $Z = Z^T \geq 0$, $W = W^T \geq 0$, any matrices N_i $(i = 1, \cdots, 4)$, S and V with appropriate dimensions, and scalars $\lambda_i > 0$ $(i = 1, 2)$ such that the following LMI holds.

$$\begin{bmatrix} \Theta_{11} + \lambda_1 HH^T & \Theta_{12} & \Theta_{13} & \Theta_{14} & \tau P_{22} & -\tau N_1 & t_1 SE_a^T & t_1 SE_{ad}^T \\ \Theta_{12}^T & \Theta_{22} & \Theta_{23} & \Theta_{24} & \tau P_{12} & -\tau N_2 & t_2 SE_a^T & t_2 SE_{ad}^T \\ \Theta_{13}^T & \Theta_{23}^T & \Theta_{33} + \lambda_2 HH^T & \Theta_{34} & -\tau P_{22} & -\tau N_3 & t_3 SE_a^T & t_3 SE_{ad}^T \\ \Theta_{14}^T & \Theta_{24}^T & \Theta_{34}^T & \Theta_{44} & -\tau CP_{12} & -\tau N_4 & t_4 SE_a^T & t_4 SE_{ad}^T \\ \tau P_{22} & \tau P_{12}^T & -\tau P_{22} & -\tau P_{12}^T C^T & -\tau W & 0 & 0 & 0 \\ -\tau N_1^T & -\tau N_2^T & -\tau N_3^T & -\tau N_4^T & 0 & -\tau Z & 0 & 0 \\ t_1 E_a S^T & t_2 E_a S^T & t_3 E_a S^T & t_4 E_a S^T & 0 & 0 & -\lambda_1 I & 0 \\ t_1 E_{ad} S^T & t_2 E_{ad} S^T & t_3 E_{ad} S^T & t_4 E_{ad} S^T & 0 & 0 & 0 & -\lambda_2 I \end{bmatrix} < 0, \tag{20}$$

where Θ_{ij} $(i = 1, \cdots, 4; i \le j \le 4)$ are defined in (17). Moreover, a stabilizing control law is given by $u(t) = VS^{-T}x(t)$.

Proof: Replacing A and A_d in (17) with $A + HF(t)E_a$ and $A_d + HF(t)E_{ad}$, respectively, we find that (17) for Σ is equivalent to the following condition.

$$\begin{array}{l}\Theta + \Omega_{ha}^T F(t)\Omega_{ea} + \Omega_{ea}^T F^T(t)\Omega_{ha} \\ \quad +\Lambda_{hd}^T F(t)\Lambda_{ed} + \Lambda_{ed}^T F^T(t)\Lambda_{hd} < 0,\end{array} \tag{21}$$

where

$$\begin{aligned}
\Omega_{ha} &= \begin{bmatrix} -H^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Omega_{ea} &= \begin{bmatrix} t_1E_aS^T & t_2E_aS^T & t_3E_aS^T & t_4E_aST^T & 0 & 0 \end{bmatrix}, \\
\Lambda_{hd} &= \begin{bmatrix} 0 & 0 & -H^T & 0 & 0 & 0 \end{bmatrix}, \\
\Lambda_{ed} &= \begin{bmatrix} t_1E_{ad}S^T & t_2E_{ad}S^T & t_3E_{ad}S^T & t_4E_{ad}S^T & 0 & 0 \end{bmatrix}.
\end{aligned}$$

By Lemma 1, a sufficient condition guaranteeing (21) is that there exist scalars $\lambda_i > 0$ $(i = 1, 2)$ such that

$$\Theta + \lambda_1 \Omega_{ha}^T\Omega_{ha} + \lambda_1^{-1}\Omega_{ea}^T\Omega_{ea} + \lambda_2\Lambda_{hd}^T\Lambda_{hd} + \lambda_2^{-1}\Lambda_{ed}^T\Lambda_{ed} < 0. \tag{22}$$

Applying the Schur complement shows that (22) is equivalent to (20). ■

Remark 4: The optimal values of the tuning parameters t_i $(i = 1, \cdots, 4)$ that were introduced in Theorems 3 and 4 can be found by the approach stated in Remark 5 of [16]. A numerical solution to this problem can be obtained by using a numerical optimization algorithm, such as `fminsearch` in the Optimization Toolbox ver. 2.2 of Matlab 6.5.

V. Numerical Examples

The following two examples demonstrate that the above methods are an improvement over some previous ones. The first concerns the asymptotic and robust stability of a neutral system, and the second concerns the design of a state feedback controller.

Example 1: Consider the following uncertain neutral system, Σ.

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$
$$C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \qquad 0 \le c < 1,$$

where $\Delta A(t)$ and $\Delta A_d(t)$ are unknown matrices satisfying $\|\Delta A(t)\| \le \alpha$ and $\|\Delta A_d(t)\| \le \alpha$.

This system has the form of (2) with $H = I$ and $E_a = E_{ad} = \alpha I$. This example was fully discussed in [4]. In our method, all the free matrices are determined by solving the corresponding LMIs.

Table I lists the maximum upper bound, τ, for $\alpha = 0$. It is clear that the method in this paper produces significantly better results than [4] or [9], especially when c is large. It can also be seen that the parameterized matrix transformation, Corollary 2, is almost equivalent to Theorem 1; but that it becomes conservative when $c = 0$.

Table II gives τ for $\alpha = 0.2$ and different c's. For comparison, the calculation results from [4] are also listed. Clearly, Theorem 2 yields a larger τ for any c.

In addition, Table III shows what effect the uncertainty bound, α, has on τ as regards stability. The calculations are based on Theorem 2 and Han's method [4]. It can be seen that τ decreases as α increases, as mentioned in [4], and that our method yields a larger τ than Han's method.

TABLE I

Maximum upper bound, τ, on constant time delay for $\alpha = 0$ (nominal system).

c	0	0.1	0.3	0.5	0.7	0.9
Fridman's paper [9]	4.47	3.49	2.06	1.14	0.54	0.13
Corollary 1	4.47	3.65	2.32	1.31	0.57	0.10
Han's paper [4]	4.35	4.33	4.10	3.62	2.73	0.99
Corollary 2	4.37	4.35	4.13	3.67	2.87	1.41
Theorem 1	4.47	4.35	4.13	3.67	2.87	1.41

TABLE II

Maximum upper bound, τ, on constant time-delay for $\alpha = 0.2$ (uncertain system).

c	0	0.05	0.1	0.15	0.2
Han's paper [4]	1.77	1.63	1.48	1.33	1.16
Theorem 2	2.43	2.33	2.24	2.14	2.03
c	0.25	0.3	0.35	0.4	
Han's paper [4]	0.98	0.79	0.59	0.37	
Theorem 2	1.91	1.78	1.65	1.50	

TABLE III

Effect of uncertainty bound, α, on τ for $c = 0.1$.

α	0	0.05	0.1	0.15	0.2	0.25
Han's paper [4]	4.33	3.61	2.90	2.19	1.48	0.77
Theorem 2	4.35	3.64	3.06	2.60	2.24	1.94

Remark 5: In [5], Han and Yu employed the discretized-Lyapunov-functional method to obtain less conservative results. However, their method is difficult to extend to the synthesis of a controller.

Example 2: Consider the uncertain system Σ with

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \; A_d = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$H = I, \; E_a = 0.2I, \; E_{ad} = \alpha I.$$

α was chosen to be 0.2 in [15] and 0 in [16]. The maximum upper bound, τ, for which the system is stabilized by state feedback was found to be 0.4500 in the former and 0.5865 in the latter ($\alpha = 0$ in [16]). For $\alpha = 0.2$, Theorem 4 yields $\tau = 0.6548$ when $t_1 = 1, t_2 = 0.8, t_3 = 0$, and $t_4 = 0$; and the corresponding state feedback gain is $K = [-24.5739 \; -17.6699]$. Also, for $\alpha = 0$, $\tau = 0.9518$ and $K = [-24.8739 \; -10.8616]$ when $t_1 = 1, t_2 = 1.2, t_3 = 0$, and $t_4 = 0$.

For system stabilization, our method yields a larger τ than previous ones, mainly because it combines the free-weighting-matrix method with a parameterized model transformation, thus garnering the advantages of both.

VI. Conclusion

In this study, instead of representing the delayed term as an integral, free weighting matrices are used to express the relationships between the terms in the Leibniz-Newton formula. In order to use the parameterized-matrix method, a new parameterized-matrix form is presented. These two methods are combined to obtain a new stability criterion for a nominal neutral system. The free weighting matrices and the parameter matrix are easily determined by solving an LMI. The criterion thus obtained is further extended to a neutral system with time-varying structured uncertainties. These stability criteria are employed to derive a stabilizing state feedback controller.

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