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A Fuzzy Control Strategy for Acrobots Combining Model-Free and Model-Based Control

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Short title: A Fuzzy Control Strategy for Acrobots

Abstract: This paper describes a fuzzy control strategy for the control of an acrobot. The strategy combines model-free and model-based fuzzy control. The model-free fuzzy controller designed for the upswing ensures that the energy of the acrobot increases with each swing. The control law for the torque is derived directly from the energy of the acrobot. The model-based fuzzy controller, which is based on a Takagi-Sugeno fuzzy model for balancing, employs the concept of parallel distributed compensation. The stability of the fuzzy control system for balance control is guaranteed by a common symmetric positive matrix, which satisfies linear matrix inequalities and is found by a convex optimization technique.

Key words: acrobot, underactuated mechanical systems, fuzzy control, Takagi-Sugeno fuzzy model, linear matrix inequality.

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1. Introduction

Underactuated mechanical systems possess fewer actuators than the degrees of freedom [1]. This kind of system can perform complex tasks with a small number of actuators and has the advantages of being light, cheap, energy-efficient, and highly reliable. For these reasons, underactuated mechanical systems have been receiving a great deal of attention recently. On the other hand, because of the complexity of their nonlinear dynamics and their holonomic/nonholonomic behavior, control of this kind of system is very difficult [e.g., 2, 3]

An acrobot is a good example of an underactuated mechanical system. The acrobot considered in this paper is a two-link manipulator operating in a vertical plane. It consists of one joint each at the shoulder and elbow with a single actuator at the elbow. The first link, which is attached to the passive joint, can rotate freely. The second link is attached to the actuated joint, where a motor is mounted to provide a control torque. The control objective in this study is to swing it up from the stable downward equilibrium position to the unstable straight-up equilibrium position and balance it there.

In recent years, a considerable number of approaches and methods of controlling an acrobot have been proposed. For example, Hauser and Murray [4], and Bortoff and Spong [5] have investigated the problem of balancing an acrobot at the unstable straight-up equilibrium position and controlling its motion along the unstable equilibrium manifold using nonlinear-approximation and pseudolinearization methods, respectively. Berkemeier and Fearing [6] have studied the application of nonlinear control to achieve sliding and hopping gaits of an acrobot. A time-state control scheme has been proposed by She, et al. [7] for upswing control.

Lee and Smith [8] have described a fuzzy control method that combines genetic algorithms, dynamic switching fuzzy systems and meta-rule techniques for the automatic design and tuning of an acrobot fuzzy control system. The genetic algorithms utilize PD control results. They showed that the performance was much better than that obtainable with PD control. However, this method is very complicated.

In [9], Brown and Passino have presented the design of an LQR (linear quadratic regulator), fuzzy and adapitive fuzzy controllers to balance the acrobot, and a PD controller with inner-loop partial feedback linearization, a state feedback controller and a fuzzy controller to swing the acrobot up. They also used genetic algorithms to tune the parameters of the balancing and swing-up controllers. The simulation results were good. However, the methods are complicated and the settling time is relatively long.

Spong [10, 11, 12] has described a partial feedback linearization method to swing an acrobot up and has used the techniques of pseudolinearization/LQR to balance it. The basic swing-up strategy is to choose an external control to swing the second link so that the amplitude of the swing of the first link increases with each swing. However, the upswing control law was chosen based on the condition under which the energy of a *single* link increases. So, theoretically, it did not guarantee that the energy of the acrobot increased with each swing. In addition, as pointed out by [11], the LQR balancing control law makes the region for balance control very small.

This paper proposes a fuzzy control strategy that employs a model-free fuzzy controller to swing the acrobot up and a model-based fuzzy controller to balance it. In the swing-up process, the control law for the torque is derived directly from the energy of the acrobot, and the model-free fuzzy controller regulates the amplitude of the control torque according to the energy. The key point is to choose a control torque that guarantees that the energy of the acrobot increases with each swing. This is quite different from the method proposed by Spong [10, 11, 12]. The main feature of this strategy is that the amplitude of the control torque decreases as the energy increases. Hence, the acrobot moves into the neighborhood of the unstable straight-up equilibrium position very smoothly. In the balancing process, a Takagi-Sugeno fuzzy model is constructed to approximate the dynamics of the acrobot. The model-based fuzzy controller, which uses the Takagi-Sugeno fuzzy model, employs the concept of parallel distributed compensation. Unlike the methods of [8, 9], the design is simple, and the stability of the fuzzy control system for balance control is guaranteed by a common symmetric positive matrix, which satisfies linear matrix inequalities (LMIs) and is found by a convex optimization technique. Since the Takagi-Sugeno fuzzy model describes the acrobot with a satisfactory approximated precision over a large region, the model-based fuzzy balancing control law makes the region for balance control larger than it is with LQR. The validity of the proposed strategy is demonstrated by simulation results.

2. Dynamics of the acrobot

Consider the acrobot shown in Figure 1. Its dynamic equations are

$$m_{11}(\mathbf{q})\ddot{q}_1 + m_{12}(\mathbf{q})\ddot{q}_2 + c_1(\mathbf{q},\ \dot{\mathbf{q}}) + g_1(\mathbf{q}) = 0\,, \tag{1a}$$

$$m_{21}(\mathbf{q})\ddot{q}_1 + m_{22}(\mathbf{q})\ddot{q}_2 + c_2(\mathbf{q},\ \dot{\mathbf{q}}) + g_2(\mathbf{q}) = \tau\,, \tag{2b}$$

where

$$\mathbf{q} = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^T, \tag{2}$$

$$m_{11}(\mathbf{q}) = m_1 L_{g1}^2 + I_1 + m_2 L_{g2}^2 + I_2 + m_2 L_1^2 + 2 m_2 L_1 L_{g2} \cos q_2\,, \tag{3a}$$

$$m_{22}(\mathbf{q}) = m_2 L_{g2}^2 + I_2\,, \tag{3b}$$

$$m_{12}(\mathbf{q}) = m_{21}(\mathbf{q}) = m_2 L_{g2}^2 + I_2 + m_2 L_1 L_{g2} \cos q_2\,, \tag{3c}$$

$$c_1(\mathbf{q},\ \dot{\mathbf{q}}) = -m_2 L_1 L_{g2} \dot{q}_2 (2\dot{q}_1 + \dot{q}_2) \sin q_2\,, \tag{4a}$$

$$c_2(\mathbf{q},\ \dot{\mathbf{q}}) = m_2 L_1 L_{g2} \dot{q}_1^2 \sin q_2\,, \tag{4b}$$

$$g_1(\mathbf{q}) = -(m_1 L_{g1} + m_2 L_1) g \sin q_1 - m_2 L_{g2} g \sin(q_1 + q_2)\,, \tag{5a}$$

$$g_2(\mathbf{q}) = -m_2 L_{g2} g \sin(q_1 + q_2)\,. \tag{5b}$$

For the link $i\,(i=1,\,2)$, the parameters q_i, $\dot{q}_i$, m_i, L_i, L_{gi} and I_i are the angle, the angular velocity, the mass, the link length, the center of mass, and the moment of inertia, respectively. The inertia matrix $\mathbf{M}(\mathbf{q})$ is

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_{11}(\mathbf{q}) & m_{12}(\mathbf{q}) \\ m_{21}(\mathbf{q}) & m_{22}(\mathbf{q}) \end{bmatrix}, \tag{6}$$

which is symmetric and positive definite.

[Insert Figure 1 about here]

The acrobot has the following characteristics:

1) It is second-order nonholonomic.

2) It cannot be exactly linearized in the time domain.

Remark: Characteristic 1 is direct result of Proposition 2.1 and 2.2 in [1]. Characteristic 2 can easily be derived from Lemma 2.5 in [13].

In this paper, the motion space of the acrobot is divided into two subspaces: one is the attractive area in the neighborhood of the unstable straight-up equilibrium position, and the remainder is the swing-up area. Two small positive numbers, λ_1 and λ_2, are used to define the two subspaces.

$$\text{Swing-up area: } |q_1| > \lambda_1 \text{ or } |q_1 + q_2| > \lambda_2, \tag{7}$$

$$\text{Attractive area: } |q_1| \le \lambda_1 \text{ and } |q_1 + q_2| \le \lambda_2. \tag{8}$$

In the swing-up area, a model-free fuzzy controller swings the acrobot up; in the attractive area, a fuzzy controller based on a Takagi-Sugeno fuzzy model balances it.

3. Control in the swing-up area

In the swing-up area, the control torque is derived directly from the energy of the acrobot. A model-free fuzzy controller is designed to regulate its amplitude in order to guarantee smooth movement from the swing-up area into the attractive area.

3.1. Determining the control torque

The energy of the acrobot is given by

$$E(\mathbf{q},\ \dot{\mathbf{q}}) = T(\mathbf{q},\ \dot{\mathbf{q}}) + V(\mathbf{q}), \tag{9}$$

where $T(\mathbf{q},\ \dot{\mathbf{q}})$ is the kinetic energy and $V(\mathbf{q})$ is the potential energy, both of which are expressed in generalized coordinates. They are defined as follows:

$$T(\mathbf{q},\ \dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}, \tag{10a}$$

$$V(\mathbf{q}) = \sum_{i=1}^{2} V_i(\mathbf{q}) = \sum_{i=1}^{2} m_i g h_i(\mathbf{q}), \tag{10b}$$

where $V_i(\mathbf{q})$ and $h_i(\mathbf{q})$ are the potential energy and the height of the center of mass of the ith link, respectively. $h_1(\mathbf{q})$ and $h_2(\mathbf{q})$ are given by

$$h_1(\mathbf{q}) = (m_1 L_{g1} + m_2 L_1) g \cos q_1, \tag{11a}$$

$$h_2(\mathbf{q}) = m_2 L_{g2} g \cos(q_1 + q_2). \tag{11b}$$

During an upswing, the energy of the acrobot should increase continuously until it reaches the amount that the acrobot has at the unstable straight-up equilibrium position. This means that the derivation of the energy should satisfy the following condition in the swing-up area.

$$\dot{E}(\mathbf{q},\ \dot{\mathbf{q}}) \geq 0. \tag{12}$$

Differentiating (9) yields

$$\dot{E}(\mathbf{q},\ \dot{\mathbf{q}}) = \begin{bmatrix} \frac{\partial T(\mathbf{q},\ \dot{\mathbf{q}})}{\partial \dot{q}_1} & \frac{\partial T(\mathbf{q},\ \dot{\mathbf{q}})}{\partial \dot{q}_2} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} \frac{\partial T(\mathbf{q},\ \dot{\mathbf{q}})}{\partial q_1} & \frac{\partial T(\mathbf{q},\dot{\mathbf{q}})}{\partial q_2} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{\partial V(\mathbf{q})}{\partial q_1} & \frac{\partial V(\mathbf{q})}{\partial q_2} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}. \tag{13}$$

From (10a), we obtain

$$\begin{bmatrix} \frac{\partial T(\mathbf{q},\ \dot{\mathbf{q}})}{\partial \dot{q}_1} & \frac{\partial T(\mathbf{q},\ \dot{\mathbf{q}})}{\partial \dot{q}_2} \end{bmatrix} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 \end{bmatrix} \mathbf{M}(\mathbf{q}), \tag{14}$$

and

$$\begin{bmatrix} \frac{\partial T(\mathbf{q},\ \dot{\mathbf{q}})}{\partial q_1} & \frac{\partial T(\mathbf{q},\ \dot{\mathbf{q}})}{\partial q_2} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} c_1(\mathbf{q},\ \dot{\mathbf{q}}) & c_2(\mathbf{q},\ \dot{\mathbf{q}}) \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}. \tag{15}$$

The following equation is derived from (10b)

$$\begin{bmatrix} \frac{\partial V(\mathbf{q})}{\partial q_1} & \frac{\partial V(\mathbf{q})}{\partial q_2} \end{bmatrix} = \begin{bmatrix} g_1(\mathbf{q}) & g_2(\mathbf{q}) \end{bmatrix}. \tag{16}$$

Rewriting the dynamic equations (1a) and (1b) gives

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \mathbf{M}^{-1}(\mathbf{q}) \begin{bmatrix} -c_1(\mathbf{q},\ \dot{\mathbf{q}}) - g_1(\mathbf{q}) \\ \tau - c_2(\mathbf{q},\ \dot{\mathbf{q}}) - g_2(\mathbf{q}) \end{bmatrix}. \tag{17}$$

Substituting (14), (15), (16), and (17) into (13) yields

$$\dot{E}(\mathbf{q},\ \dot{\mathbf{q}}) = \dot{q}_2 \tau. \tag{18}$$

So, the control torque for swing-up may be chosen to be

$$\tau = \mathrm{sgn}(\dot{q}_2)\upsilon, \qquad \upsilon \geq 0 \tag{19}$$

to satisfy (12).

3.2 . Design of model-free fuzzy controller

The control variable υ in (19) can be chosen arbitrarily in the admissible range of the control torque as long as it is positive. Clearly, the amplitude of the control torque should be chosen so that it decreases as the energy increases. That makes the acrobot enter the attractive area smoothly when the control law changes. To implement this strategy, a model-free fuzzy controller is designed to determine the control variable υ. Since the only function of the model-free fuzzy control is to ensure that the amplitude of the control torque decreases when the energy increases, a simple fuzzy controller is good enough.

A basic fuzzy control method [14, 15] is used to design the model-free fuzzy controller. The input of the model-free fuzzy controller is the energy $E(\mathbf{q}, \dot{\mathbf{q}})$, the fuzzy output variable is υ_f, and the crisp output is the control variable υ. The fuzzy relation between the energy $E(\mathbf{q}, \dot{\mathbf{q}})$ and the fuzzy output variable υ_f is the set of simple fuzzy rules listed in Table 1. The membership functions (mfs) for fuzzy input/output linguistic variables are chosen to have the triangular shapes shown in Figure 2. The crisp output, υ, is obtained by applying the center-of-gravity defuzzification method to the fuzzy output variable υ_f.

[Insert Table 1 about here]

[Insert Figure 2 about here]

Remark: The fuzzy rules listed in Table 1 can also be implemented with a simple linear controller with saturation. However, fuzzy logic gives us flexibility in constructing a control law. If two suitable fuzzy sets for the energy, $E(\mathbf{q}, \dot{\mathbf{q}})$, and the fuzzy output variable, υ_f, are chosen, it is possible to obtain a control law that provides better control performance, e.g., a shorter settling time, smoother movement from the swing-up area into the attractive area, etc., using fuzzy logic rather than a simple linear controller.

The model-free fuzzy controller is employed until the acrobot enters the attractive area. Then the fuzzy controller based on a Takagi-Sugeno fuzzy model balances it.

4. Control in the attractive area

The attractive area is defined as $q_1 \in [-\lambda_1, \lambda_1]$ and $q_1 + q_2 \in [-\lambda_2, \lambda_2]$. The dynamics of the acrobot in this area is nonlinear, and a linear approximate model around the unstable straight-up equilibrium position is usually used for control. However, linearization based solely on the coordinate of the unstable straight-up equilibrium position makes either the attractive area very small, or the control torque for the acrobot entering the area very large. For example, if λ_1 and λ_2 are both $\pi/4$, then q_2 is in the range $[-\lambda_1 - \lambda_2, \lambda_1 + \lambda_2] = [-\pi/2, \pi/2]$. Clearly, $\cos(q_2)$, which is involved in the dynamics, cannot be approximated very well over such a wide range. To achieve better control, a model is needed that describes the motion in this area more precisely.

Takagi and Sugeno [16] have introduced a model-based analytical method into fuzzy control (Takagi-Sugeno fuzzy model). The main feature of a Takagi-Sugeno fuzzy model is that the local dynamics of each fuzzy implication are described by a linear model. The overall fuzzy model of the system is a fuzzy blend of the linear models. The Takagi-Sugeno fuzzy modeling method is a multiple model approach that can handle uncertain and time-varying situations. To design a model-based fuzzy controller, a set of fuzzy rules is first used to derive suitable local linear state space models. Then, a set of local controllers is designed based on the models using the parallel distributed compensation method. Finally, the fuzzy controller is obtained by the fuzzy blending of the local controllers. This method gives us a more suitable way to describe the nonlinearity of the acrobot in the attractive area. The dynamics of the acrobot in this area are captured by a set of fuzzy implications that characterize local relations. When the fuzzy controller obtained by the fuzzy blending of the local controllers is used to balance the acrobot, the stability of the fuzzy control system for balance control is guaranteed if a common symmetric positive definite matrix can be found for all local linear models.

4.1. Takagi-Sugeno fuzzy model

To reduce the design effort and complexity, as few rules as possible are chosen. The Takagi-Sugeno fuzzy system in the attractive area is shown in Table 2, where $z = |q_1|/|q_2|$, $\mathbf{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [q_1 \quad q_2 \quad \dot{q}_1 \quad \dot{q}_2]^T$, c_1 and c_2 are constants, and $c_1 > c_2 > 0$.

[Insert Table 2 about here]

It is clear that two linear models are used to describe the acrobot. In Rule 1, the acrobot is linearized with the coordinate $\mathbf{x}_\delta = [0 \quad \delta \quad 0 \quad 0]^T$; and in Rule 2, it is linearized with the coordinate $\mathbf{x}_\theta = [0 \quad \theta \quad 0 \quad 0]^T$ (where $\theta > \delta \geq 0$).

In the attractive area, $q_1 \in [-\lambda_1, \lambda_1]$ and $q_1 + q_2 \in [-\lambda_2, \lambda_2]$. Since λ_1 and λ_2

are very small, $\sin q_1$ and $\sin(q_1+q_2)$ can be approximated by q_1 and q_1+q_2, respectively. According to equations (1a) and (1b), the linear approximate model for the coordinate $\mathbf{x}_\phi = \begin{bmatrix} 0 & \phi & 0 & 0 \end{bmatrix}^T$ (where $\mathbf{x}_\phi = \mathbf{x}_\delta$ or $\mathbf{x}_\theta$) is as follows:

$$\dot{\mathbf{x}} = \mathbf{A}(\phi)\mathbf{x} + \mathbf{b}(\phi)\tau \,, \tag{20}$$

where

$$\mathbf{A}(\phi) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31}(\phi) & a_{32}(\phi) & 0 & 0 \\ a_{41}(\phi) & a_{42}(\phi) & 0 & 0 \end{bmatrix}, \ \mathbf{b}(\phi) = \begin{bmatrix} 0 \\ 0 \\ b_3(\phi) \\ b_4(\phi) \end{bmatrix}, \tag{21}$$

$$\mathbf{q}_\phi = \begin{bmatrix} 0 & \phi \end{bmatrix}^T, \tag{22a}$$

$$\begin{bmatrix} a_{41}(\phi) & -a_{42}(\phi) & b_4(\phi) \\ -a_{31}(\phi) & a_{32}(\phi) & -b_3(\phi) \end{bmatrix} = \frac{\mathbf{M}(\mathbf{q}_\phi)}{\det \mathbf{M}(\mathbf{q}_\phi)} \begin{bmatrix} -\beta & \beta & 1 \\ \alpha+\beta & -\beta & 0 \end{bmatrix}, \tag{22b}$$

$$\alpha = -(m_1 L_{g1} + m_2 L_1) g \,, \tag{23a}$$

$$\beta = -m_2 L_{g2} g \,, \tag{23b}$$

Substituting the coordinates $\mathbf{x}_\delta$ and $\mathbf{x}_\theta$ into (21) yields the following two local linear models

$$(\mathbf{A}_1,\ \mathbf{b}_1) = (\mathbf{A}(\delta),\ \mathbf{b}(\delta)) \tag{24}$$

and

$$(\mathbf{A}_2,\ \mathbf{b}_2) = (\mathbf{A}(\theta),\ \mathbf{b}(\theta)) \,, \tag{25}$$

respectively. So, the dynamics of the approximate fuzzy model is represented by

$$\dot{\mathbf{x}} = \frac{\sum_{j=1}^{2} \mu_j(z)(\mathbf{A}_j \mathbf{x} + \mathbf{b}_j \tau)}{\sum_{j=1}^{2} \mu_j(z)} \,, \tag{26}$$

where $\mu_1(z)$ and $\mu_2(z)$ are the membership functions for Rules 1 and 2, respectively. They are defined as

$$\mu_1(z) = \begin{cases} 0 & 0 \le z \le c_2 \\ \frac{1}{2} + \frac{1}{2}\sin\frac{\pi}{c_1 - c_2}\left(z - \frac{c_1 + c_2}{2}\right) & c_2 < z \le c_1 \\ 1 & z > c_1 \end{cases} \tag{27a}$$

$$\mu_2(z) = 1 - \mu_1(z) \,, \tag{27b}$$

and shown in Figure 3.

[Insert Figure 3 about here]

4.2. Design of fuzzy controller

The concept of parallel distributed compensation [17, 18] is utilized to design local controllers. The basic idea is to design a corresponding controller for each local linear model. This paper employs the pole assignment approach to design the local linear controllers. The full state is assumed to be available and the design results are given in Table 3. Finally, the resulting overall fuzzy controller obtained by the fuzzy blending of the individual linear controllers is

$$\tau = \frac{-\sum_{j=1}^{2} \mu_j(z)\mathbf{f}_j\mathbf{x}}{\sum_{j=1}^{2} \mu_j(z)}. \tag{28}$$

This is used to balance the acrobot. Controller (28) is nonlinear in general. It is clear that the parallel distributed compensation method employs two controllers with automatic switching via fuzzy rules.

[Insert Table 3 about here]

Substituting (28) into (26) yields the following fuzzy control system:

$$\dot{\mathbf{x}} = \frac{\sum_{j=1}^{2}\sum_{k=1}^{2} \mu_j(z)\mu_k(z)(\mathbf{A}_j - \mathbf{b}_j\mathbf{f}_k)\mathbf{x}}{\sum_{j=1}^{2}\sum_{k=1}^{2} \mu_j(z)\mu_k(z)}. \tag{29}$$

To guarantee stability, the results in [19] were applied to the fuzzy control system (29), and the following sufficient condition for stability was obtained.

Theorem 1: The fuzzy control system (29) is asymptotically stable at the unstable straight-up equilibrium position if there exists a common symmetric positive definite matrix $\mathbf{P}$ such that the following LMIs hold:

$$(\mathbf{A}_j - \mathbf{b}_j\mathbf{f}_k)^T\mathbf{P} + \mathbf{P}(\mathbf{A}_j - \mathbf{b}_j\mathbf{f}_k) < 0, \qquad j, k = 1, 2 \tag{30}$$

It is known that finding the matrix $\mathbf{P}$ is a convex feasibility problem. Great efforts have been devoted to solving this problem. A trial-and-error procedure (Tanaka and Sugeno, 1992) has been tried. Now, this problem can be solved efficiently by using the interior-point method [20].

5. Simulation

The parameters of the acrobot are given in Table 4 [11]. The parameters λ_1 and λ_2, which divide the motion space, are chosen to be

$$\lambda_1 = \lambda_2 = \pi/4 \text{ rad}. \tag{31}$$

Two linear approximate models in the attractive area are obtained at

$$\delta = 0 \text{ rad} \tag{32a}$$

and

$$\theta = \pi / 4 \text{ rad}. \tag{32a}$$

The parameters c_1 and c_2, which are used to construct the Takagi-Sugeno fuzzy model, are chosen to be

$$c_1 = 4, \quad c_2 = 0.1. \tag{33}$$

[Insert Table 4 about here]

For the attractive area, substituting δ, θ and the parameters in Table 4 into (24) and (25) yields these local linear models:

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 12.6163 & -12.6797 & 0 & 0 \\ -14.7325 & 29.5830 & 0 & 0 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 0 \\ 0 \\ -3.0147 \\ 6.0332 \end{bmatrix}, \tag{34}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 9.9163 & -5.4432 & 0 & 0 \\ -7.8176 & 15.7068 & 0 & 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ 0 \\ -1.6003 \\ 3.2029 \end{bmatrix}. \tag{35}$$

Two local controllers are designed by applying the method of parallel distributed compensation to $(\mathbf{A}_1, \mathbf{b}_1)$ and $(\mathbf{A}_2, \mathbf{b}_2)$. The local feedback gains $\mathbf{f}_1$ and $\mathbf{f}_2$ are determined by selecting (-2, –2.4, –2.2, –2.6) as the eigenvalues of the local linear subsystems. They are:

$$\mathbf{f}_1 = [-72.5571 \quad -24.0169 \quad -30.5870 \quad -13.7590], \tag{36}$$

$$\mathbf{f}_2 = \left[-134.8258 \quad -49.486 \quad -53.998 \quad -24.108\right]. \tag{37}$$

The overall parallel distributed compensation controller is

$$\tau = -\mu_1(z)\mathbf{f}_1\mathbf{x} - \mu_2(z)\mathbf{f}_2\mathbf{x}. \tag{38}$$

The following symmetric positive definite matrix $\mathbf{P}$ is obtained by using the LMI algorithm.

$$\mathbf{P} = \begin{bmatrix} 8.1223 & 3.4031 & 3.2487 & 1.5189 \\ 3.4031 & 1.4306 & 1.3710 & 0.6416 \\ 3.2487 & 1.3710 & 1.9326 & 0.9003 \\ 1.5189 & 0.6416 & 0.9003 & 0.4210 \end{bmatrix}. \tag{39}$$

So, the sufficient condition for stabilizing (30) is satisfied. In other words, the fuzzy control system is asymptotically stable for fuzzy control law (38).

If we let the energy of the acrobot in the horizontal position be zero, then the energy at the unstable straight-up equilibrium position is 24.5 J, and the energy range is [-24.5 J, 24.5 J]. If we assume that the maximum torque is 3 Nm, then the range of the control torque is [-3 Nm, 3 Nm].

Figures 4-9 show simulation results for the initial condition $\mathbf{x}(0) = \begin{bmatrix} \pi & 0 & 0 & 0 \end{bmatrix}^T$. When $0 \le t < 7.63$ s, the model-free fuzzy controller is used for swing-up control. The energy keeps increasing and the amplitude of the control torque keeps decreasing during this period. The control law is switched from model-free fuzzy control to model-based fuzzy control at $t = 7.63$ s. The model-based fuzzy controller is used for balancing control when $t \ge 7.63$ s. The simulation results show that the response is very soft when the control law changes, and the control torque in the attractive area is very small; the state converges smoothly to the unstable straight-up equilibrium position.

[Insert Figures 4-9 about here]

6. Conclusions

A control strategy combining model-free and model-based fuzzy control has been developed for controlling an acrobot. The model-free fuzzy controller is used for swing-up control. It is designed to guarantee that the energy of the acrobot increases with each swing, and the amplitude of the control torque decreases as the energy increases. This strategy ensures a soft switching of the control laws when the acrobot passes from the swing-up area into the attractive area. The model-based fuzzy controller is used for balance control and is designed by combining the Takagi-Sugeno fuzzy model with the method of parallel distributed compensation. The stability of the fuzzy control system for balance control is guaranteed by a common symmetric positive matrix. Simulation results have demonstrated the validity of the method.

The proposed strategy can easily be extended to the motion control of an *n*-degree-of-freedom underactuated mechanical system in a vertical plane. It can also be used on some mechanical systems with strong nonlinearity, for example, an inverted pendulum.

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Captions of Figures and Tables

Figure 1. Model of the acrobot.
Figure 2. Membership functions of the input/output linguistic variable.
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Table 1. Fuzzy control rules to swing the acrobot up.
Table 2. Takagi-Sugeno fuzzy system for the acrobot.
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Table 4. Parameters of the acrobot for simulation.

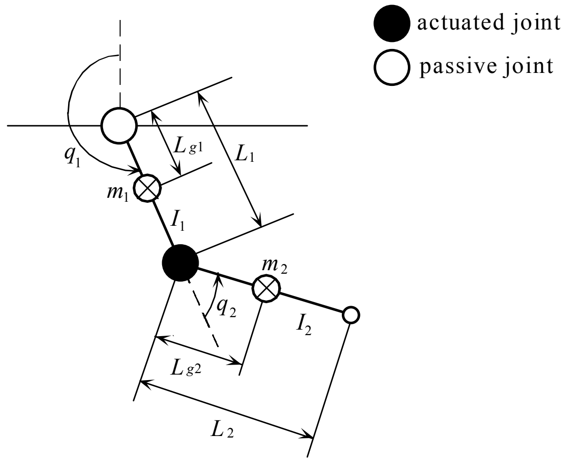

Figure 1. Model of the acrobot.

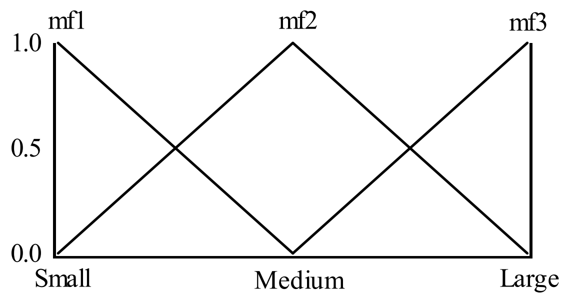

Figure 2. Membership functions of the input/output linguistic variable.

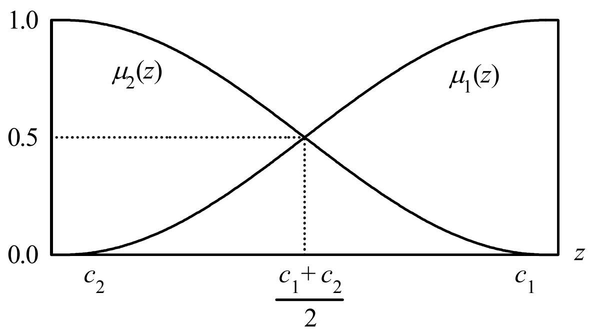

Figure 3. Membership functions $\mu_1(z)$ and $\mu_2(z)$.

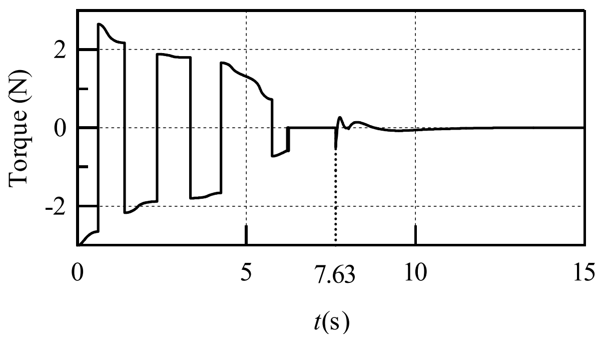

Figure 4. Control torque.

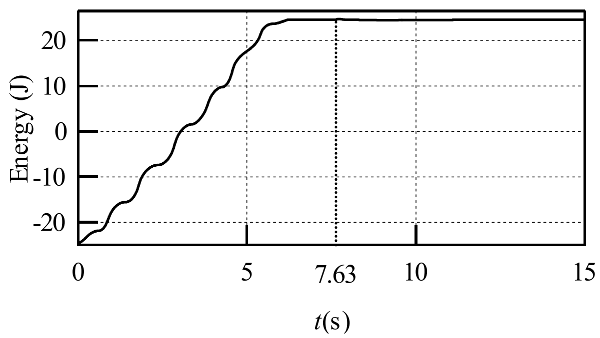

Figure 5. Energy of the acrobot.

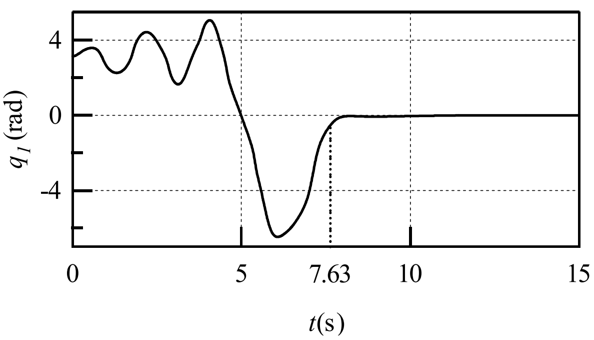

Figure 6. Angle of the first link.

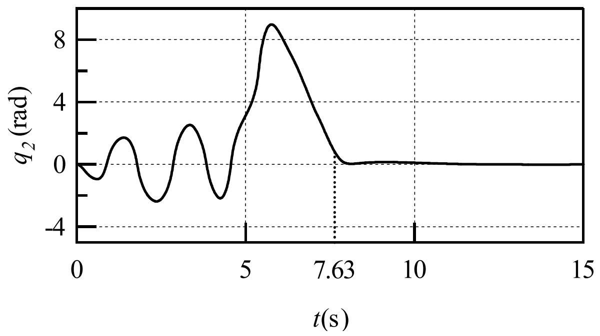

Figure 7. Angle of the second link.

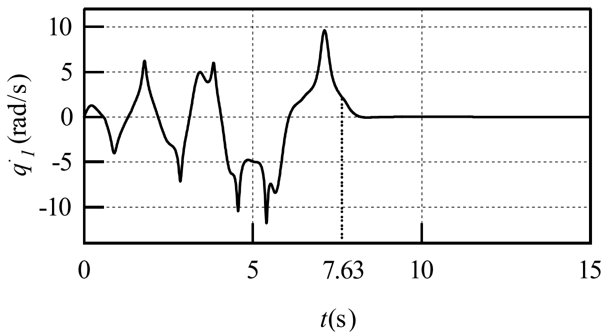

Figure 8. Velocity of the first link.

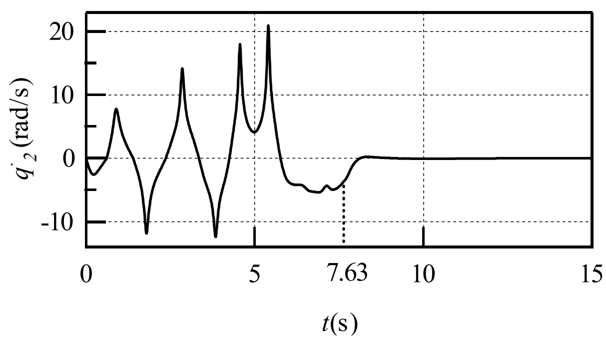

Figure 9. Velocity of the second link.

Table 1. Fuzzy control rules to swing the acrobot up.

If	Then
$E(\mathbf{q},\ \dot{\mathbf{q}})$ is small	υ_f is large
$E(\mathbf{q},\ \dot{\mathbf{q}})$ is medium	υ_f is medium
$E(\mathbf{q},\ \dot{\mathbf{q}})$ is large	υ_f is small

Table 2. Takagi-Sugeno fuzzy system for the acrobot.

Rule	If	Then
1	z is larger than c_1	$\dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{b}_1\tau$
2	z is smaller than c_2	$\dot{\mathbf{x}} = \mathbf{A}_2\mathbf{x} + \mathbf{b}_2\tau$

Table 3. The local linear controllers.

Rule	If	Then
1	z is larger than c_1	$\tau = -\mathbf{f}_1\mathbf{x}$
2	z is smaller than c_2	$\tau = -\mathbf{f}_2\mathbf{x}$

Table 4. Parameters of the acrobot for simulation.

	m_i [kg]	L_i [m]	L_{gi} [m]	I_i [Nm^2]
Link 1	1	1	0.5	0.083
Link 2	1	2	1	0.33