# Rejection of Non-periodic Disturbances in Repetitive Control Systems

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**Abstract:** This paper proposes a new approach to disturbance estimation based on a curvature model to improve the rejection performance of disturbances in a repetitive control system. The main feature is that the stability of the repetitive control system is guaranteed when the estimated disturbance is incorporated directly into the designed repetitive control law. Simulation results show that disturbances are rejected efficiently when this approach is used.

**Keywords:** Repetitive control, circle of curvature, disturbance estimation, disturbance rejection.

## 重复控制系统中非周期扰动的抑制

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摘要：为了改善重复控制系统中的扰动抑制性能，本文提出一种基于曲率模型的扰动估计新方法．该方法的主要特点是扰动的估计值直接与重复控制规则结合也不影响重复控制系统的稳定性．仿真结果显示该方法可以有效地抑制扰动．

关键词：重复控制；曲率圆；扰动估计；扰动抑制

## 1. Introduction

Control systems that perform operations cyclically are subject to both periodic and non-periodic disturbances. For example, in noncircular cutting with a lathe, the position of the cutting tool has to track a given periodic signal. Such a system is affected by random disturbances and disturbances caused by the structure of the system. Repetitive control is a very useful strategy for tracking periodic reference inputs and rejecting periodic disturbances[1], [2]. However, a repetitive control system cannot readily reject non-periodic disturbances[3].

In this paper, a new approach to disturbance estimation based on a curvature model is proposed to improve the performance of non-periodic disturbance rejection in a repetitive control system. The characteristics of this method are that disturbances are reproduced satisfactorily even though the estimation model is very simple; the stability of the system is guaranteed when disturbance estimation is directly incorporated into the repetitive control law; and no *a priori* information about disturbances, such as the pick value, is needed.

## 2. Disturbance estimation

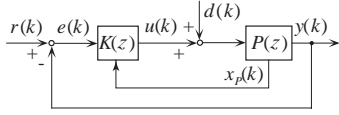The configuration of a conventional repetitive control system is shown in Fig. 1. The plant and the

Fig. 1. Conventional repetitive control system.

repetitive controller are respectively given by

$$\begin{cases} x_P(k+1) = A_P x_P(k) + B_P[u(k)+d(k)], \\ y(k) = C_P x_P(k), \end{cases} \quad (1)$$

and

$$\begin{cases} x_K(k+1) = A_K x_K(k) + A_{KP} x_P(k) + B_K e(k), \\ u(k) = C_K x_K(k) + C_{KP} x_P(k) + D_K e(k), \end{cases} \quad (2)$$

where $x_P(k) \in \mathbf{R}^{n_P}$, $x_K(k) \in \mathbf{R}^{n_K}$, $y(k) \in \mathbf{R}$, $u(k) \in \mathbf{R}$ and $d(k) \in \mathbf{R}$ are the states of the plant, repetitive controller, output, control input and non-periodic disturbances, respectively. We assume that the repetitive controller has been designed so that the internal stability of the repetitive control system is guaranteed, and also make the following assumptions.

*Assumption 1*: $(A_P, B_P)$ is controllable.

*Assumption 2*: The disturbance $d(k)$ is bounded and smooth enough.

Since an internal model of periodic signals, $1/(1-z^{-L})$, is contained in the repetitive controller, $K(z)$, perfect tracking and disturbance rejection is obtained for these signals. However, for non-periodic disturbances, good rejection performance can no longer be expected. Generally speaking, the peak value of the tracking error is proportional to the peak value of the disturbance. If some *a priori* information about disturbances, e.g. the pick value, is known, a nonlinear control law can be designed to reject the disturbances[4]. In this paper, we do not use such *a priori* information. The only assumption about the
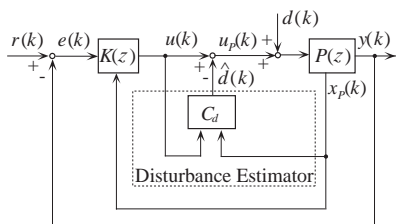


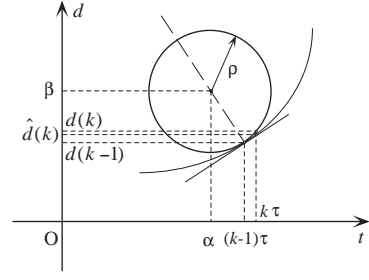Fig. 2. Proposed repetitive control system.



Fig. 3. Curvature circle model for disturbance estimation.

disturbances is that the sampling frequency is high enough that the disturbances are smooth enough.

Haskara, *et. al.* [5] have proposed a method that uses a linear model to estimate a disturbance. However, to obtain an accurate estimate might require a model with a very high order, and the calculations might be very complicated. To achieve better non-periodic disturbance rejection, Smith and Tomizuka [6] added a disturbance observer to a conventional repetitive control system; but the design is not simple. In this paper, a low-order nonlinear disturbance-estimation model called a curvature circle model is proposed for the estimation of disturbances, and is used to reduce the tracking error. The proposed repetitive control system is shown in Fig. 2.

Curvature circle approximation, which is a good way to approximate a curve, is used to estimate the disturbance. If the curvature circle at *k*-1 is known, then the value on this circle at *k* can be taken as an estimate of the disturbance at *k* (see Fig. 3). This estimate has the following characteristics:

1) The curvature circle shares the same tangent with the disturbance at *k*-1.

2) The curvature circle has the same concavity or convexity as the disturbance at *k*-1.

3) The curvature of the curvature circle equals that of the disturbance at *k*-1.

So, the characteristics of the disturbance are reflected in the estimate; and by making use of them, the disturbance can effectively be suppressed. The details are given below.

According to Assumption 1, there exists a nonsingular matrix $T \in \mathbf{R}^{n_P \times n_P}$ that converts the plant (1) into the following controllable canonical form:

$$\begin{cases} \bar{x}_P(k+1) = \bar{A}_P \bar{x}_P(k) + \bar{B}_P[u(k) + d(k)], \\ y(k) = \bar{C}_P \bar{x}_P(k), \end{cases} \quad (3)$$

where

$$\bar{A}_P = T^{-1}A_P T = \begin{bmatrix} 0 & & 1 & & \\ \vdots & & & \ddots & \\ 0 & & & & 1 \\ \hline -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n_P} \end{bmatrix},$$

$$\bar{B}_P = T^{-1}B_P = \begin{bmatrix} 0 & \cdots & 0 & | & 1 \end{bmatrix}^T,$$

$$\bar{C}_P = C_P T = \begin{bmatrix} c_1 & c_2 & \cdots & c_{n_P} \end{bmatrix}.$$

Multiplying both sides of (3) by $\bar{B}_P^T$ gives

$$\begin{cases} \bar{B}_P^T \bar{x}_P(k+1) = D\bar{x}_P(k) + u(k) + d(k), \\ D := \begin{bmatrix} -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n_P} \end{bmatrix} \end{cases} \quad (4)$$

So, the disturbance $d(k)$ can be expressed as

$$d(k) = \bar{B}_P^T \bar{x}_P(k+1) - D\bar{x}_P(k) - u(k), \quad (5)$$

and the following equations hold:

$$\begin{cases} d(k-1) = \bar{B}_P^T \bar{x}_P(k) - D\bar{x}_P(k-1) - u(k-1), \\ d(k-2) = \bar{B}_P^T \bar{x}_P(k-1) - D\bar{x}_P(k-2) - u(k-2), \\ d(k-3) = \bar{B}_P^T \bar{x}_P(k-2) - D\bar{x}_P(k-3) - u(k-3). \end{cases} \quad (6)$$

For a sampling period, $\tau$, if the first and second derivatives of the disturbance $d(k)$ at $k$-1 are approximated by

$$\begin{cases} \hat{d}'(k-1) = \dfrac{d(k-1) - d(k-2)}{\tau} \\ \hat{d}''(k-1) = \dfrac{d(k-1) - 2d(k-2) + d(k-3)}{\tau^2} \end{cases} \quad (7)$$

then the radius of the curvature circle, $\rho$, is

$$\rho^2 = \frac{[1 + \hat{d}'^2(k-1)]^3}{\hat{d}''^2(k-1)}, \quad (8)$$

and the coordinates of the center of the curvature circle are

$$\begin{cases} \alpha = (k-1)\tau - \dfrac{\hat{d}'(k-1)[1 + \hat{d}'^2(k-1)]}{\hat{d}''(k-1)}, \\ \beta = d(k-1) + \dfrac{1 + \hat{d}'^2(k-1)}{\hat{d}''(k-1)}. \end{cases} \quad (9)$$

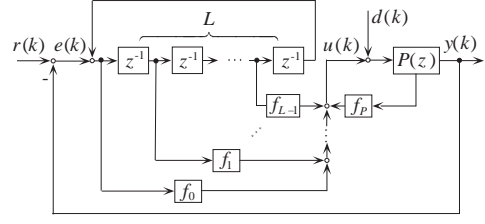Thus, the disturbance estimate $\hat{d}(k)$ is obtained from



Fig. 4. Optimal repetitive control system.

the following lemma.

*Lemma 1.* The disturbance estimate $\hat{d}(k)$ is given by

$$\hat{d}(k) = \begin{cases} \beta - \sqrt{\rho^2 - (k\tau - \alpha)^2}; & \hat{d}''(k-1) > 0, \\ d(k-1) + \tau \hat{d}'(k-1); & \hat{d}''(k-1) = 0, \\ \beta + \sqrt{\rho^2 - (k\tau - \alpha)^2}; & \hat{d}''(k-1) < 0, \end{cases} \quad (10)$$

where $\rho$, $\alpha$ and $\beta$ are given by (8) and (9).

Combining the designed repetitive control law with the disturbance estimate yields the control law

$$u_P(k) = u(k) - \hat{d}(k). \quad (11)$$

The following theorem holds for this law.

*Theorem 1.* The control law (11) guarantees the stability of the repetitive control system and suppresses disturbances when the sampling period, $\tau$, is small enough.

*Proof:* Omitted.

## 3. Numerical example

Consider the following second-order plant:

$$\begin{cases} P(s) = \dfrac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}, \\ \omega = 1\,\text{rad/s}; \quad \zeta = 0.5. \end{cases} \quad (12)$$

The sampling period and the number of steps of the repetitive controller are chosen to be

$$\tau = 0.1\,\text{s}, \quad L = 21, \quad (13)$$

respectively. The periodic reference input

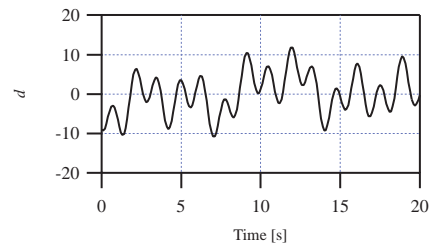$$r(k) = \sin\frac{2\pi}{21}k + \sin\frac{4\pi}{21}k \quad (14)$$



Fig. 5. Disturbances.

is added. First, choosing $Q = \begin{bmatrix} 100 \times I_{21} & 0 \\ 0 & I_2 \end{bmatrix}$, and

optimizing the following performance index

$$J := \sum_{k=0}^{\infty} [x_e^T(k)Qx_e(k) + u_e^2(k)],$$

$$x_e(k) := \begin{bmatrix} e(k) & \cdots & e(k-L+1) & (1-z^{-L})x_P^T(k) \end{bmatrix}^T,$$

$$u_e(k) := (1-z^{-L})u(k)$$

gives the optimal repetitive control law

$$u_e(k) = F_G x_e(k) = \begin{bmatrix} f_0 & \cdots & f_{L-1} & f_P \end{bmatrix} x_e(k). \quad (15)$$

The optimal repetitive control system is shown in Fig. 4 [7].

The disturbance

$$d(k) = -5\cos\frac{3\pi}{21}k - 4\sin\frac{3\pi}{50}k - 3\cos\frac{3\pi}{110}k$$
$$- 2\sin\frac{3\pi}{230}k - \cos\frac{3\pi}{410}k. \quad (16)$$

which is non-periodic up to 27 s, was input (Fig. 5). The simulation results for the optimal system are shown in Fig. 6. In the steady state, the peak-to-peak value of the tracking error is about 1. The estimates of the disturbances obtained by the method proposed in this paper and the estimation error are shown in Fig. 7. It is clear from the figure that the estimates reproduce the disturbances satisfactorily. The simulation results for a control law that makes use of the estimates are shown in Fig. 8. A comparison of Figs. 6 and 8 reveals that the addition of disturbance estimation significantly reduces the tracking error.

## 4. Conclusions

This paper proposes a curvature circle approximation model for disturbance estimation to improve the rejection performance for non-periodic disturbances. Unlike other approaches, we do not
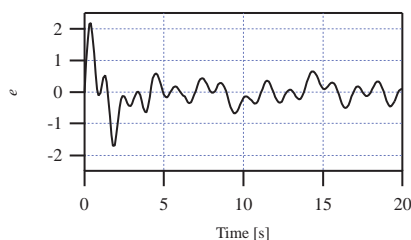


Fig. 6. Response of optimal repetitive control system.
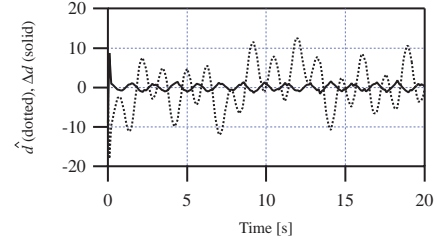


Fig. 7. Estimates of the disturbances and the estimation error.
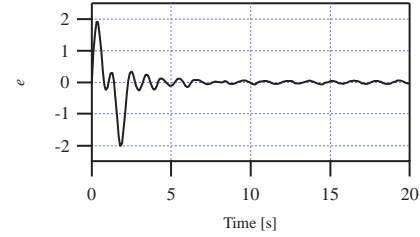


Fig. 8. Response of optimal repetitive control system with disturbance estimation.

assume that any information about the disturbances, such as the pick value, is known. The main features of this method are: 1) the disturbances are reproduced satisfactorily even though the estimation model is very simple; and 2) the stability of the repetitive control system is guaranteed when disturbance estimation is incorporated directly into the designed repetitive control law. The validity of the proposed method has been demonstrated through simulations.

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