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Flow and Temperature Control of a Tank System by Backstepping Method

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ABSTRACT
The backstepping method was applied to a tank system to control the flow and temperature at the exit. A backstepping control law and a robust backstepping control law were developed for the plant without and with uncertainties, respectively. The designed control system is asymptotically stable for the plant without uncertainties and globally uniformly bounded for the plant with uncertainties.

1. INTRODUCTION

Over the past few years, a considerable number of studies have been devoted to a new control design methodology: *backstepping* ([1]). Unlike feedback linearization, backstepping can avoid the cancellation of useful nonlinearities. So, it offers the prospect of a more practicable nonlinear control law.

This paper describes the application of the backstepping control strategy to a tank system to control the flow and temperature at the exit. Mathematical models of the system are first derived. Then, a backstepping controller is designed for the nominal plant. However, there are usually some uncertainties in the mathematical model of the plant. To achieve robustness for the control system, a robust backstepping controller is designed by improving the backstepping control law to guarantee global uniform boundedness.

Nomenclature

q_i	Rate of inflow of the water (m^3/s)
q_o	Rate of outflow of the water (m^3/s)
θ_i	Temperature of the inflow (K)
θ_o	Temperature of the outflow (K)
θ_a	Air temperature (K)
h	Height of the water level (m)
A	Cross-sectional area of the tank (m^2)
Φ_i	Heat supplied (W)
R	Equivalent heat resistance of the tank (K/W)
a	Discharge coefficient of valve $(m^{2.5}/s)$
ρ	Density of water (kg/m^3)
c_p	Specific heat of water $(J/kg/K)$

2. MATHEMATICAL MODEL OF THE TANK SYSTEM

The tank system studied here is shown in Fig. 1. In this system, cold water is sent to the tank from a waterworks. The water is heated in the tank, and then sent out. The system has two control inputs: the rate of inflow and the heater supply. The rate and temperature of the inflow, the rate of the outflow, and the temperature of the water in the tank are known.

The rate of outflow is given by

$$\frac{dh}{dt} = \frac{q_i - q_o}{A}, \tag{1}$$

$$q_o = a\sqrt{h}, \tag{2}$$

and

$$q_o \geq q_m > 0. \tag{3}$$

Let Φ_T be the heat that the water in the tank possesses, Φ_o be the heat that the output water takes away, Φ_S be the heat released to the air and Φ_c be the heat that the inflow brings in, and assume that the temperature of the water in the tank is uniform. Then we have

$$\Phi_T = -\Phi_o - \Phi_s + \Phi_c + \Phi_i. \tag{4}$$

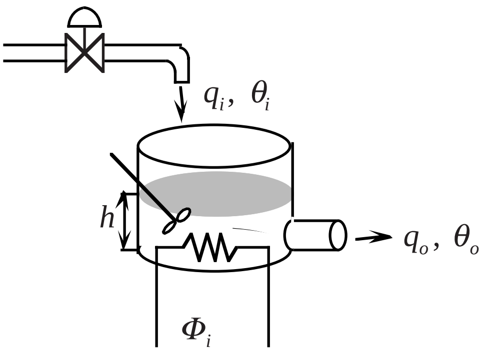

FIGURE 1: The tank system.

and

$$\Phi_T = \frac{A\rho c_p}{a^2} q_o^2 \frac{d\theta_o}{dt}, \tag{5}$$

$$\Phi_o = c_p \rho \theta_o q_o, \tag{6}$$

$$\Phi_s = \frac{\theta_o - \theta_a}{R[h, A]} = \frac{\theta_o - \theta_a}{R[q_o]}, \tag{7}$$

$$\Phi_c = c_p \rho \theta_i q_i . \tag{8}$$

From the above relationships, the model of the tank can be written as

$$\begin{cases} \dfrac{dq_o}{dt} = -\dfrac{a^2}{2A} + \dfrac{a^2}{2A}\dfrac{1}{q_o} q_i \\ \dfrac{d\theta_o}{dt} = -B(\dfrac{c_p\rho}{q_o} + \dfrac{1}{q_o^2 R})\theta_o + \dfrac{B\theta_a}{q_o^2 R} \\ \qquad + \dfrac{Bc_p\rho\theta_i}{q_o^2} q_i + \dfrac{B}{q_o^2}\Phi_i, \end{cases} \tag{9}$$

where

$$B = a^2/(Ac_p\rho).$$

It is clear from (9) that the tank system is a two-input two-output nonlinear system. If we let $\bar{q}_o$ and $\bar{\theta}_o$ be the desired outputs, then the control objective is to make the outflow and temperature track them. In the design of such control systems, the flow sub-system and the temperature sub-system are usually considered separately, and the correlation between the controlled outputs is ignored. Linear control theory is mainly used for control system design. In particular, PID controllers are generally designed for each linearized sub-system ([2]).

In this study, we used the MIMO nonlinear model directly so as to take the nonlinearties of the plant and the correlation between the controlled outputs into account. Decomposing the outputs yields

$$\begin{cases} q_o = \bar{q}_o + q_{oe} \\ \theta_o = \bar{\theta}_o + \theta_{oe}, \end{cases} \tag{10}$$

where q_{oe} and θ_{oe} are the errors between the real and the desired outputs. If we substitute (10) into (9) and transform the control inputs into

$$\begin{cases} q_i = (\bar{q}_o + q_{oe})(1 + u_q) \\ \Phi_i = \{(\bar{q}_o + q_{oe})c_p\rho + \dfrac{1}{R}\}\bar{\theta}_o - \dfrac{\theta_a}{R} \\ \qquad -(\bar{q}_o + q_{oe})c_p\rho\theta_i + u_\Phi, \end{cases} \tag{11}$$

then the plant model becomes

$$\begin{cases} \dfrac{dq_{oe}}{dt} = \dfrac{a^2}{2A} u_q \\ \dfrac{d\theta_{oe}}{dt} = -\dfrac{Bc_p\rho\theta_{oe}}{\bar{q}_o + q_{oe}} - \dfrac{B\theta_{oe}}{(\bar{q}_o + q_{oe})^2 R} \\ \qquad + \dfrac{Bc_p\rho\theta_i}{\bar{q}_o + q_{oe}} u_q + \dfrac{B}{(\bar{q}_o + q_{oe})^2} u_\Phi . \end{cases} \tag{12}$$

If we let

$$x := \begin{bmatrix} q_{oe} & \theta_{oe} \end{bmatrix}^T, \tag{13}$$

and rewrite the model in the form:

$$dx/dt = f(x) + g_1(x)u_q + g_2(x)u_\Phi, \tag{14}$$

then

$$\begin{cases} f(x) = \begin{bmatrix} 0 \\ -\dfrac{Bc_p\rho\theta_{oe}}{\bar{q}_o + q_{oe}} - \dfrac{B\theta_{oe}}{(\bar{q}_o + q_{oe})^2 R} \end{bmatrix} \\ g_1(x) = \begin{bmatrix} \dfrac{a^2}{2A} \\ \dfrac{Bc_p\rho\theta_i}{\bar{q}_o + q_{oe}} \end{bmatrix} \quad g_2(x) = \begin{bmatrix} 0 \\ \dfrac{B}{(\bar{q}_o + q_{oe})^2} \end{bmatrix}. \end{cases} \tag{15}$$

(14) is the nominal model of the plant. Since it is hard to obtain an exact model of a real plant, it is useful to include uncertainties in the plant model as follows:

$$\begin{cases} dx/dt = f(x) + g_1(x)u_q + g_2(x)u_\Phi + \bar{\Delta} \\ \bar{\Delta} = \begin{bmatrix} \dfrac{a^2}{2A}\dfrac{1}{\bar{q}_o + q_{oe}}\Delta_1 & \dfrac{B}{(\bar{q}_o + q_{oe})^2}\Delta_2 \end{bmatrix}^T, \end{cases} \tag{16}$$

with

$$|\Delta_1| \le \delta_1, \quad |\Delta_2| \le \delta_2 . \tag{17}$$

3. DESIGN OF BACKSTEPPING CONTROL LAWS

We first consider a plant without uncertainties. If a Lyapunov function is defined as

$$V(x) := \frac{1}{2} x^T x = \frac{1}{2} q_{oe}^2 + \frac{1}{2} \theta_{oe}^2 , \tag{18}$$

it is clear that the following control law

$$\begin{bmatrix} u_q \\ u_\Phi \end{bmatrix} := \begin{bmatrix} \alpha_q(x) \\ \alpha_\Phi(x) \end{bmatrix} = \begin{bmatrix} g_1(x) & g_2(x) \end{bmatrix}^{-1} \begin{bmatrix} -k_q q_{oe} \\ -k_\theta \theta_{oe} \end{bmatrix}$$
$$= \begin{bmatrix} -k_q \dfrac{2A}{a^2} q_{oe} \\ -\dfrac{1}{B}(\bar{q}_o + q_{oe})[2k_q\theta_i q_{oe} + k_\theta(\bar{q}_o + q_{oe})\theta_{oe}] \end{bmatrix} \tag{19}$$

makes $\frac{dV}{dt} = \frac{\partial V}{\partial x}(f + g_1\alpha_q + g_2\alpha_\Phi)$ negative definite if

$$k_q, k_\theta > 0 \tag{20}$$

are chosen.

Now, backstepping the plant (14) gives

$$\begin{cases} dx/dt = f(x) + g_1(x)v_q + g_2(x)v_\Phi \\ \dfrac{d}{dt}\begin{bmatrix} v_q \\ v_\Phi \end{bmatrix} = \begin{bmatrix} u_q \\ u_\Phi \end{bmatrix}. \end{cases} \tag{21}$$

A new Lyapunov function is selected:

$$V_a := V(x) + \frac{1}{2} z_q^2 + \frac{1}{2} z_\Phi^2 , \tag{22}$$

where

$$\begin{cases} z_q := v_q - \alpha_q(x) \\ z_\Phi := v_\Phi - \alpha_\Phi(x) \end{cases}. \tag{23}$$

Then we have the following lemma.
Lemma 1: The control law

$$u_q = -(k_1 k_q \frac{2A}{a^2} + \frac{a^2}{2A}) q_{oe} - (k_1 + k_q) v_q - \frac{B c_p \rho \theta_i}{\bar{q}_o + q_{oe}} \theta_{oe} , \tag{24-1}$$

$$\begin{aligned} u_\Phi = &-k_2\{v_\Phi + \frac{1}{B}(\bar{q}_o + q_{oe})[2k_q\theta_i q_{oe} \\ &+k_\theta(\bar{q}_o + q_{oe})\theta_{oe}]\} - c_p\rho\{k_q\theta_i(\bar{q}_o + 2q_{oe}) \\ &+k_\theta(\bar{q}_o + q_{oe})\theta_{oe}\}v_q + k_\theta c_p \rho\theta_{oe}(\bar{q}_o + q_{oe}) \\ &+k_\theta\{\frac{\theta_{oe}}{R} - c_p\rho\theta_i(\bar{q}_o + q_{oe})v_q - v_\Phi\} \\ &-\frac{B\theta_{oe}}{(\bar{q}_o + q_{oe})^2} \end{aligned} \tag{24-2}$$

guarantees the global asymptotic stability of the system (21) at $(q_o, \theta_o) = (\bar{q}_o, \bar{\theta}_o)$ for any $k_1, k_2 > 0$.
Proof:

$$\begin{aligned} &dV_a/dt = dV/dt + (v_q - \alpha_q)(u_q - d\alpha_q/dt) \\ &+(v_\Phi - \alpha_\Phi)(u_\Phi - d\alpha_\Phi/dt) \\ &= \frac{\partial V}{\partial x}(f + g_1\alpha_q + g_2\alpha_\Phi + g_1 z_q + g_2 z_\Phi) \\ &+z_q[u_q - \frac{\partial\alpha_q}{\partial x}(f + g_1 v_q + g_2 v_\Phi)] \\ &+z_\Phi[u_\Phi - \frac{\partial\alpha_\Phi}{\partial x}(f + g_1 v_q + g_2 v_\Phi)]. \end{aligned}$$

So, if the control law is chosen to be

$$\begin{aligned} u_q &= -k_1(v_q - \alpha_q) + \frac{\partial\alpha_q}{\partial x}(f + g_1 v_q + g_2 v_\Phi) - \frac{\partial V}{\partial x} g_1 \\ &= -k_1(v_q + k_q \frac{2A}{a^2} q_{oe}) - k_q v_q - \frac{a^2}{2A} q_{oe} - \frac{B c_p \rho\theta_i}{\bar{q}_o + q_{oe}}\theta_{oe} \\ &= -(k_1 k_q \frac{2A}{a^2} + \frac{a^2}{2A}) q_{oe} - (k_1 + k_q) v_q - \frac{B c_p \rho\theta_i}{\bar{q}_o + q_{oe}}\theta_{oe} , \end{aligned}$$

$$\begin{aligned} u_\Phi &= -k_2(v_\Phi - \alpha_\Phi) + \frac{\partial\alpha_\Phi}{\partial x}(f + g_1 v_q + g_2 v_\Phi) - \frac{\partial V}{\partial x} g_2 \\ &= -k_2\{v_\Phi + \frac{1}{B}(\bar{q}_o + q_{oe})[2k_q\theta_i q_{oe} + k_\theta(\bar{q}_o + q_{oe})\theta_{oe}]\} \\ &-c_p\rho\{k_q\theta_i(\bar{q}_o + 2q_{oe}) + k_\theta(\bar{q}_o + q_{oe})\theta_{oe}\}v_q \\ &+k_\theta c_p\rho\theta_{oe}(\bar{q}_o + q_{oe}) \\ &+k_\theta\{\frac{\theta_{oe}}{R} - c_p\rho\theta_i(\bar{q}_o + q_{oe})v_q - v_\Phi\} - \frac{B\theta_{oe}}{(\bar{q}_o + q_{oe})^2} , \end{aligned}$$

then

$$\frac{dV_a}{dt} = \frac{\partial V}{\partial x}(f + g_1\alpha_q + g_2\alpha_\Phi) - k_1 z_q^2 - k_2 z_\Phi^2 < 0 .$$

That guarantees the global asymptotic stability at $(q_{oe}, \theta_{oe}) = (0, 0)$, i.e. $(q_o, \theta_o) = (\bar{q}_o, \bar{\theta}_o)$. (QED)

Based on the above backstepping control law, a robust backstepping control law is derived for a plant with uncertainties (16):

Lemma 2: The control law

$$\begin{cases} u_q = -(k_1 k_q \frac{2A}{a^2} + \frac{a^2}{2A}) q_{oe} - (k_1 + k_q) v_q \\ \quad - \frac{Bc_p \rho \theta_i}{\overline{q}_o + q_{oe}} \theta_{oe} - k_{\Delta 1} k_q \,\mathrm{sgn}[z_q] \frac{\delta_1}{\overline{q}_o + q_{oe}}, \\ u_\Phi = -k_2 \{ v_\Phi + \frac{1}{B}(\overline{q}_o + q_{oe})[2k_q \theta_i q_{oe} \\ \quad + k_\theta (\overline{q}_o + q_{oe})\theta_{oe}]\} - c_p \rho \{ k_q \theta_i (\overline{q}_o + 2q_{oe}) \\ \quad + k_\theta (\overline{q}_o + q_{oe})\theta_{oe} \} v_q + k_\theta c_p \rho (\overline{q}_o + q_{oe})\theta_{oe} \\ \quad + k_\theta \{ \frac{\theta_{oe}}{R} - c_p \rho \theta_i (\overline{q}_o + q_{oe}) v_q - v_\Phi \} \\ \quad - \frac{B\theta_{oe}}{(\overline{q}_o + q_{oe})^2} \\ \quad - k_{\Delta 2} \,\mathrm{sgn}[z_\Phi] \{ (c_p \rho k_q \theta_i |q_{oe} + 2q_{oe}| \\ \quad + k_\theta (\overline{q}_o + q_{oe}) |\theta_{oe}|) \frac{1}{\overline{q}_o + q_{oe}} \delta_1 + k_\theta \delta_2 \} \end{cases} \tag{25}$$

guarantees the global uniform boundedness of the system:

$$\begin{cases} dx/dt = f(x) + g_1(x) v_q + g_2(x) v_\Phi + \overline{\Delta} \\ \frac{d}{dt}\begin{bmatrix} v_q \\ v_\Phi \end{bmatrix} = \begin{bmatrix} u_q \\ u_\Phi \end{bmatrix} \end{cases} \tag{26}$$

at $(q_o, \theta_o) = (\overline{q}_o, \overline{\theta}_o)$ for any $\sqrt{k_q - \frac{1}{q_m^2}}, \sqrt{k_\theta - \frac{1}{q_m^4}} > 0$, $k_1, k_2 > 0$ and $k_{\Delta 1}, k_{\Delta 2} > 0$.

Proof: Defining

$$\left|\frac{\partial \alpha}{\partial x}\right| := \left[\left|\frac{\partial \alpha}{\partial x_1}\right| \quad \left|\frac{\partial \alpha}{\partial x_2}\right| \right] \ (\alpha \in R)$$

$\mathrm{sgn}(y) := [\mathrm{sgn}(y_1) \quad \mathrm{sgn}(y_2)]$ for $y = [y_1 \quad y_2]$ and

$$\overline{\delta} = \left[\frac{a^2}{2A} \frac{1}{\overline{q}_o + q_{oe}} \delta_1 \quad \frac{B}{(\overline{q}_o + q_{oe})^2} \delta_2 \right]^T$$

yields

$$dV_a/dt = dV/dt + (v_q - \alpha_q)(u_q - d\alpha_q/dt)$$
$$+ (v_\Phi - \alpha_\Phi)(u_\Phi - d\alpha_\Phi/dt)$$
$$= \frac{\partial V}{\partial x}(f + g_1 \alpha_q + g_2 \alpha_\Phi + g_1 z_q + g_2 z_\Phi) + \frac{\partial V}{\partial x}\overline{\Delta}$$
$$+ z_q \{ u_q - \frac{\partial \alpha_q}{\partial x}(f + g_1 v_q + g_2 v_\Phi) \} - z_q \frac{\partial \alpha_q}{\partial x}\overline{\Delta}$$
$$+ z_\Phi \{ u_\Phi - \frac{\partial \alpha_\Phi}{\partial x}(f + g_1 v_q + g_2 v_\Phi) \} - z_\Phi \frac{\partial \alpha_\Phi}{\partial x}\overline{\Delta}.$$

Using Young's Inequality

$$\alpha\beta \le \lambda\alpha^2 + \frac{1}{4\lambda}\beta^2,$$

where $(\alpha, \beta) \in R^2$ and λ is any positive number, yields

$$\frac{\partial V}{\partial x}\overline{\Delta} = \frac{a^2}{2A}\frac{q_{oe}}{\overline{q}_o + q_{oe}}\Delta_1 + B\frac{\theta_{oe}}{(\overline{q}_o + q_{oe})^2}\Delta_2$$
$$\le \frac{q_{oe}^2}{(\overline{q}_o + q_{oe})^2} + \frac{a^4}{16A^2}\delta_1^2 + \frac{\theta_{oe}^2}{(\overline{q}_o + q_{oe})^4} + \frac{B^2}{4}\delta_2^2$$
$$\le \frac{q_{oe}^2}{q_m^2} + \frac{\theta_{oe}^2}{q_m^4} + \frac{a^4}{16A^2}\delta_1^2 + \frac{B^2}{4}\delta_2^2.$$

So, if the control law is chosen to be

$$u_q = -k_1(v_q - \alpha_q) + \frac{\partial \alpha_q}{\partial x}(f + g_1 v_q + g_2 v_\Phi) - \frac{\partial V}{\partial x} g_1$$
$$- k_{\Delta 1} \,\mathrm{sgn}[v_q - \alpha_q] \left|\frac{\partial \alpha_q}{\partial x}\right| \overline{\delta}$$
$$= -(k_1 k_q \frac{2A}{a^2} + \frac{a^2}{2A}) q_{oe} - (k_1 + k_q) v_q$$
$$- \frac{Bc_p \rho \theta_i}{\overline{q}_o + q_{oe}} \theta_{oe} - k_{\Delta 1} k_q \,\mathrm{sgn}[z_q] \frac{\delta_1}{\overline{q}_o + q_{oe}},$$

$$u_\Phi = -k_2(v_\Phi - \alpha_\Phi) + \frac{\partial \alpha_\Phi}{\partial x}(f + g_1 v_q + g_2 v_\Phi)$$
$$- \frac{\partial V}{\partial x} g_2 - k_{\Delta 2} \,\mathrm{sgn}[v_\Phi - \alpha_\Phi] \left|\frac{\partial \alpha_\Phi}{\partial x}\right| \overline{\delta}$$
$$= -k_2 \{ v_\Phi + \frac{1}{B}(\overline{q}_o + q_{oe})[2k_q \theta_i q_{oe}$$
$$+ k_\theta (\overline{q}_o + q_{oe})\theta_{oe}]\} - c_p \rho [k_q \theta_i (\overline{q}_o + 2q_{oe})$$
$$+ k_\theta (\overline{q}_o + q_{oe})\theta_{oe}] v_q + k_\theta \{ c_p \rho \theta_{oe} (\overline{q}_o + q_{oe})$$
$$+ \frac{\theta_{oe}}{R} - c_p \rho \theta_i (\overline{q}_o + q_{oe}) v_q - v_\Phi \}$$
$$- \frac{B\theta_{oe}}{(\overline{q}_o + q_{oe})^2} - k_{\Delta 2} \,\mathrm{sgn}[z_\Phi] (\frac{(c_p \rho k_q \theta_i |\overline{q}_o + 2q_{oe}|}{\overline{q}_o + q_{oe}}$$
$$+ \frac{k_\theta (\overline{q}_o + q_{oe}) |\theta_{oe}|)}{\overline{q}_o + q_{oe}} \delta_1 + k_\theta \delta_2),$$

then

$$\frac{dV_a}{dt} = \frac{\partial V}{\partial x}(f + g_1 \alpha_q + g_2 \alpha_\Phi) - k_1 z_q^2 - k_2 z_\Phi^2$$

$$+\frac{\partial V}{\partial x}\bar{\Delta} - k_{\Delta 1}\,\mathrm{sgn}[z_q]\left|\frac{\partial \alpha_q}{\partial x}\right| z_q(\bar{\delta} + \frac{\mathrm{sgn}[z_q \frac{\partial \alpha_q}{\partial x}]}{k_{\Delta 1}}\bar{\Delta})$$

$$-k_{\Delta 2}\,\mathrm{sgn}[z_\Phi]\left|\frac{\partial \alpha_\Phi}{\partial x}\right| z_\Phi(\delta + \frac{\mathrm{sgn}[z_\Phi \frac{\partial \alpha_\Phi}{\partial x}]}{k_{\Delta 2}}\bar{\Delta})$$

$$\le -(k_q - \frac{1}{q_m^2})q_{oe}^2 - k_1 z_q^2 - (k_\theta - \frac{1}{q_m^4})\theta_{oe}^2 - k_2 z_\Phi^2$$

$$+\frac{a^4}{16A^2}\delta_1^2 + \frac{B^2}{4}\delta_2^2 .$$

That implies that dV_a / dt is negative when $\|X\| > \frac{\sqrt{\frac{a^4}{16A^2}\delta_1^2 + \frac{B^2}{4}\delta_2^2}}{\zeta}$, where $X := \left[q_{oe}\ \theta_{oe}\ z_q\ z_\Phi\right]^T$ and $\zeta := \min(\sqrt{k_q - \frac{1}{q_m^2}}, \sqrt{k_\theta - \frac{1}{q_m^4}}, \sqrt{k_1}, \sqrt{k_2})$, i.e. the state X is globally uniformly bounded. (QED)

4. NUMERICAL EXAMPLE

Let the parameters of the tank system be

$$\begin{cases} A = 0.2826\ m^2 \\ a = 8.688\times 10^{-3}\ m^{2.5}/s \\ R = 1000\ K/W \\ \theta_a = 288.15\ K = 15\ °\mathrm{C} \\ \theta_i = 289.15\ K = 16\ °\mathrm{C} \end{cases} \tag{27}$$

and

$$\begin{cases} q_o(0) = 1.3\times 10^{-3}\ m^3/s \\ \theta_o(0) = 289.15\ K = 16\ °\mathrm{C} \end{cases} \tag{28}$$

The desired outputs are

$$\begin{cases} \bar{q}_o = 1.0\times 10^{-3}\ m^3/s \\ \bar{\theta}_o = 292.15\ K = 19\ °\mathrm{C} \end{cases} \tag{29}$$

The simulation results are shown in Figs. 2 and 3.
In Fig. 2, the nominal plant is controlled by the control law (24). The parameters of the controller are

$$\begin{cases} k_q = k_\theta = 4.73\times 10^{-3} \\ k_1 = k_2 = 7.5\times 10^5 \end{cases} \tag{30}$$

It can be seen that the designed control system is stable and the outputs reach the desired values after 1000

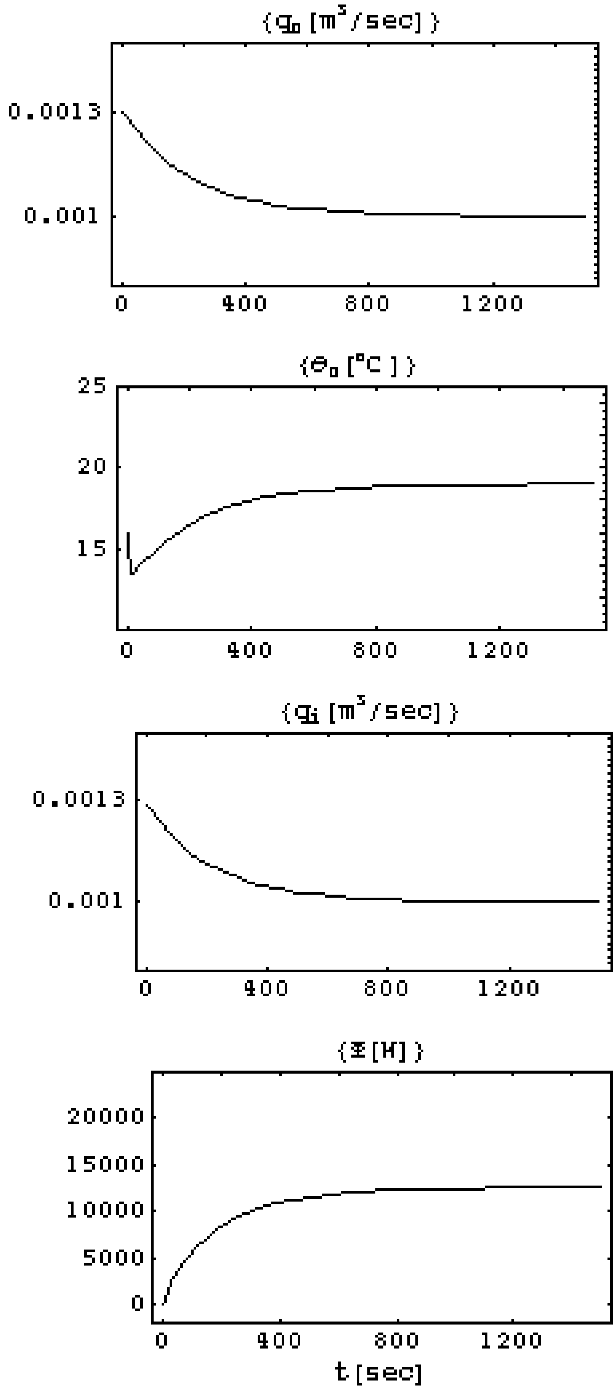

FIGURE 2: Simulation results for the nominal plant.

In Fig. 3, the plant is assumed to contain uncertainties, and Δ_1 and Δ_2 are

$$\begin{cases} \Delta_1 = 5.0\times 10^{-5}\sin(t) \\ \Delta_2 = 1.0\times 10^{-4}\sin(t) \end{cases} \tag{31}$$

The parameters of the controller (25) are chosen to be

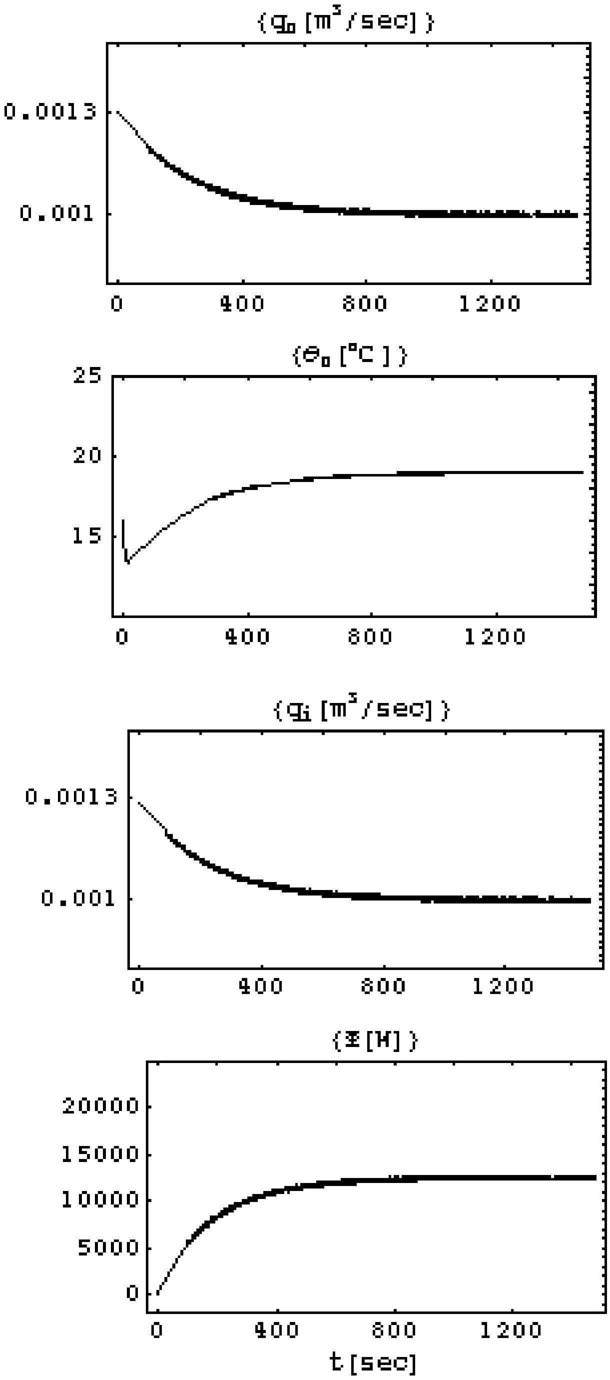

FIGURE 3: Simulation results for a plant with uncertainties.

$$k_{\Delta 1} = k_{\Delta 2} = 1.0 \tag{32}$$

The simulation results show that the designed control system is globally uniformly bounded and the outputs converge to the desired values after 1000 seconds.

5. CONCLUSIONS

In this paper, a mathematical model of a tank system is first derived, and a backstepping control law is developed for the nominal plant. Then, based on the control law, a robust backstepping control law is developed for a plant with uncertainties. The validity of these control laws is demonstrated by simulations.

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