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HIGH-PRECISION POSITIONING CONTROL BY POSITION-DEPENDENT REPETITIVE CONTROL METHOD

Jin–Hua She
Department of Mechatronics Engineering, Tokyo Engineering University,
1404–1 Katakura, Hachioji, Tokyo, 192 Japan

Michio Nakano and Takashi Naitoh
Department of Control System Engineering, Tokyo Institute of Technology,
2–12–1 Oh–Okayama, Meguro–ku, Tokyo, 152 Japan

ABSTRACT

In positioning control systems, the reference input is frequently given as a periodic function of a standard position. Within this framework, this paper introduces a new concept called the "position domain", and develops a systematic approach to the design of positioning control systems in which the speed at which the input is scanned fluctuates periodically with respect to position. A linear periodic model in the "position domain" is obtained by a transformation from the time domain. A two–degree–of–freedom (TDF) control system configuration has been employed. A repetitive controller has been designed in the "position domain" in which the output traces a position–dependent periodic reference input without steady-state error, and eliminates the effects of such fluctuations. A repetitive deadbeat controller has been designed to obtain the desired transient response.

Keywords: Position-Dependent Signals, Positioning Control, Repetitive Control, Deadbeat Control, H^∞ Control.

I INTRODUCTION

In many positioning control problems, the reference input is frequently given as a periodic function of a standard position. For instance, in a noncircular cutting process, the reference input of the cutting tool is a periodic function of the rotational angle of the spindle. To obtain high positioning precision, it has been shown that using a repetitive control system [1] , [2],[3],[4] designed in the "time domain" is very effective with the ideal case being the scanning of the input waveform at a certain constant speed. However, position–dependent disturbances caused by the system structure and machining result in fluctuations in the scanning speed. In such a case, fluctuations of the scanning speed adversely affect the reference input in the time domain. As an example, let the reference

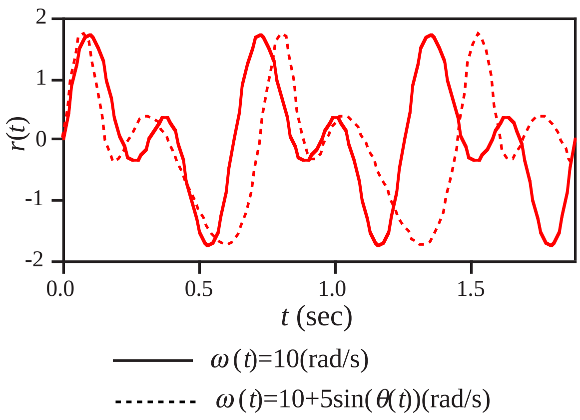

Fig.1 Variation of reference input in 'time domain' caused by scanning speed fluctuations.

input be $r(t) = \sin(\theta(t)) + \sin(2\theta(t))$. This input waveform is plotted in the time domain in Fig.1 for a constant scanning speed $\omega(t) = 10\ (rad/s)$ and for a fluctuating speed given by $\omega(t) = 10 + 5\sin(\theta(t))\ (rad/s)$. As can be seen, the scanning speed fluctuations induce variations in the period and waveform of the reference input in the time domain. For this reason, a repetitive controller designed in the time domain might not maintain a high positioning precision. So we need to develop a new design method to handle such cases.

In this paper, we consider the high-precision positioning control problem when the scanning speed fluctuates periodically with respect to a standard position. First, we introduce the concept of the "position domain", and present a transformation to obtain a model in the "position domain". Secondly, we propose a design method in the "position domain". This method yields the desired transient and steady-state response, and provides robustness with regard to scanning speed fluctuations. Finally, some experimental results are shown to demonstrate the effectiveness of the proposed approach.

Notation and Definitions

λ: delay operator ($\lambda = z^{-1}$).

$\boldsymbol{RH}_\infty$: set of all real-rational functions in λ which have no poles in the closed unit disc.

$\boldsymbol{R}[\lambda]$: ring of polynomials in λ ($\subset \boldsymbol{RH}_\infty$).

$\|G(\lambda)\|_\infty := \sup_{0\le\phi\le 2\pi} \left| G(e^{j\phi}) \right|$; $(G(\lambda) \in \boldsymbol{RH}_\infty)$

$\Omega[a(\lambda)]$: set of all zeros of the polynomial $a(\lambda)$.

Ⅱ SYSTEM MODELING IN THE "POSITION DOMAIN"

In the positioning control problem considered here, the reference input is a periodic function of a standard position. Since the reference input is based on position, it is clearly more convenient to formulate this problem in terms of position than time. The "position domain" is defined as follows [5]:

Definition: The "position domain" is a set in which every element is a function of the standard position.

To guarantee that the "position domain" is completely equivalent to the time domain, an extra condition on the transformation must be satisfied:

Transformation Condition: There exists a transformation from the time domain to the "position domain" if and only if the direction of the scanning speed is unchanged. Without loss of generality, if we use θ to denote the standard position, then the condition can be expressed as

$$\omega(t) = \frac{d\theta}{dt} > 0; \quad \forall t > 0. \tag{2.1}$$

Throughout this paper, Condition (2.1) will be assumed to be satisfied.

Consider a plant for which the state-space description in the time domain is given by

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) \tag{2.2a}$$

$$y(t) = Cx(t), \tag{2.2b}$$

where $u(t)$ is the control input, $y(t)$ is the output, and $x(t)$ is the state.

In view of (2.1), the relationship between the time domain and the "position domain" can be summarized as

$$t := t(\theta) \tag{2.3a}$$

$$\omega = \omega(t) = \omega(t(\theta)) := \tilde{\omega}(\theta) \tag{2.3b}$$

$$u = u(t) = u(t(\theta)) := \tilde{u}(\theta) \tag{2.3c}$$

$$y = y(t) = y(t(\theta)) := \tilde{y}(\theta) \tag{2.3d}$$

$$x = x(t) = x(t(\theta)) := \tilde{x}(\theta) \tag{2.3e}$$

$$\frac{dx}{dt} = \frac{d\tilde{x}(\theta)}{d\theta}\frac{d\theta}{dt} = \tilde{\omega}(\theta)\frac{d\tilde{x}(\theta)}{d\theta}. \tag{2.3f}$$

Substituting (2.3) into (2.2) yields the plant model in the "position domain":

$$\frac{d\tilde{x}(\theta)}{d\theta} = \tilde{A}\tilde{x}(\theta) + \tilde{B}\tilde{u}(\theta) \tag{2.4a}$$

$$\tilde{y}(\theta) = \tilde{C}\tilde{x}(\theta), \tag{2.4b}$$

where

$$\tilde{A} = \frac{A}{\tilde{\omega}(\theta)}; \quad \tilde{B} = \frac{B}{\tilde{\omega}(\theta)}; \quad \tilde{C} = C. \tag{2.5}$$

From (2.5) it is clear that, if the scanning speed fluctuates periodically with respect to position, then so do the system matrices $\tilde{A}$ and $\tilde{B}$. Therefore the plant in the "position domain" turns out to be a linear periodic plant with the same period.

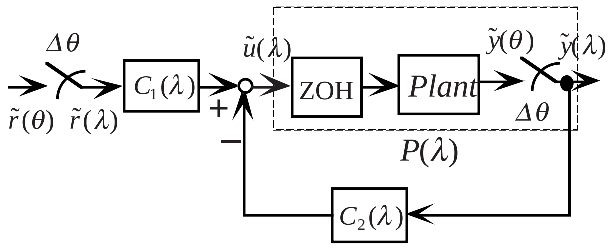

Fig. 2 Configuration of a positioning control system designed in the "position domain".

On the other hand, in the "position domain", the nominal model corresponding to the ideal case of a constant scanning speed, ω_0, can be written as

$$\frac{d\tilde{x}(\theta)}{d\theta} = \tilde{A}_0\tilde{x}(\theta) + \tilde{B}_0\tilde{u}(\theta) \tag{2.6a}$$

$$\tilde{y}(\theta) = \tilde{C}_0\tilde{x}(\theta), \tag{2.6b}$$

where

$$\tilde{A}_0 = \frac{A}{\omega_0}; \quad \tilde{B}_0 = \frac{B}{\omega_0}; \quad \tilde{C}_0 = C. \tag{2.7}$$

If we put a sampler and a hold on the output and the input , respectively, of the model (2.6), and set the sampling period to $\Delta\theta$, then we obtain the pulse–transfer function of the plant and denote it as P.

In the next section, we develop a control system design method. We initially assume that the fluctuations in the scanning speed are very small, so that they will not disrupt the internal stability of the designed control system, because we carry out our design using the nominal model (2.6) without considering the system's robust stability. However, since a real system (2.4) is linear periodic, in order to obtain a high positioning precision, we then consider how to eliminate the effects of such variations in the plant.

Ⅲ CONTROL SYSTEM DESIGN

As is well known, a L-periodic signal can be written as

$$\tilde{r}(\lambda) = \frac{\bar{r}(\lambda)}{1-\lambda^L} = \frac{r_0 + r_1\lambda + r_2\lambda^2 + \ldots + r_{L-1}\lambda^{L-1}}{1-\lambda^L}. \tag{3.1}$$

Now, let's consider the positioning control problem for such a reference input. Here, we make two assumptions. The first is for solvability and the second is for simplicity.

Assumption 1: The plant has no zeros in common with ($1-\lambda^L$).

Assumption 2: The reference input and the fluctuations of the scanning speed have the same period.

A positioning control system must satisfy the following requirements:

1) *Tracking performance:* It must provide the desired transient and steady-state input–output performance.

2) *Robustness:* The effects of scanning speed fluctuations must be eliminated.

The two-degree-of-freedom (TDF) control system configuration [6] [7] shown in Fig. 2 is used here. It is well known that a TDF configuration enables 1) and 2) to be satisfied independently.

Let

$$P=\frac{N}{D}=\frac{\lambda^m b}{a}=\frac{\lambda^m(b_0+b_1\lambda+...+b_l\lambda^l)}{a_0+a_1\lambda+...+a_n\lambda^n}$$
$$a,b,N,D\in \boldsymbol{R}[\lambda] \qquad (3.2)$$

be a coprime factorization of the plant P.

From Assumption 1, it is clear that N and $(1-\lambda^L)D$ are coprime and there exist $X,Y'\in \boldsymbol{R}[\lambda]$ such that

$$XN+(1-\lambda^L)Y'D=1. \qquad (3.3)$$

Define

$$Y:=(1-\lambda^L)Y'. \qquad (3.4)$$

Then all TDF repetitive controllers that stabilize the system can be characterized by

$$C=[C_1 \quad C_2]=(Y-K_2N)^{-1}[K_1 \quad X+K_2D]$$
$$K_1,K_2\in \boldsymbol{RH}_\infty. \qquad (3.5)$$

From Theorem 1 in [8], it is known that, if we insert a repetitive controller with a period of L in the controller C, i.e.

$$(1-\lambda^L)^{-1}(Y-K_2N)\in \boldsymbol{RH}_\infty, \qquad (3.6)$$

then in the steady-state, the effects of periodic variations in the plant will be eliminated and the output will track the reference input without steady-state error. To satisfy condition (3.6), we only need to restrict the class K_2 to be

$$K_2=(1-\lambda^L)K_2'; \quad K_2'\in \boldsymbol{RH}_\infty. \qquad (3.7)$$

So, all the TDF repetitive controllers can be parameterized as

$$C=[C_1 \quad C_2]$$
$$=(1-\lambda^L)^{-1}(Y'-K_2'N)^{-1}[K_1 \quad X+(1-\lambda^L)K_2'D]$$
$$K_1,K_2'\in \boldsymbol{RH}_\infty, \qquad (3.8)$$

where K_1 and K_2' are free parameters to be designed.

Design of Parameter K_1. The input-output transfer function in Fig. 2 is given by

$$G_{\tilde{y}\tilde{r}}=NK_1. \qquad (3.9)$$

It means that we can obtain the desired input-output performance independently of the desired closed loop transfer performance by an appropriate choice of $K_1\in \boldsymbol{RH}_\infty$.

For the desired input–output response, a deadbeat response is one possible choice. To obtain a low–ripple deadbeat response, the following conditions must be satisfied [9], [10]:

(i) *Deadbeat condition:* The tracking error between the reference input and the output must be a finite polynomial, i.e. $\tilde{e}:=\tilde{r}-\tilde{y}=\sum_{i=0}^{\mu}\tilde{e}_i\lambda^i$, where μ is a finite positive integer.

(ii) *Low–ripple condition:* The transfer function from the reference input to the control input must be a finite polynomial, i.e. $G_{\tilde{u}\tilde{r}}\in \boldsymbol{R}[\lambda]$, where $G_{\tilde{u}\tilde{r}}=DK_1$.

(i) and (ii) are satisfied if and only if we restrict K_1 to

$$K_1\in \boldsymbol{R}[\lambda]. \qquad (3.10)$$

Due to (3.8), the tracking error becomes

$$\tilde{e}=\tilde{r}-\tilde{y}=(1-NK_1)\tilde{r}=(1-NK_1)\frac{\bar{r}}{1-\lambda^L}. \qquad (3.11)$$

For this to be a finite polynomial, there must exist a finite polynomial $f\in \boldsymbol{R}[\lambda]$ such that

$$1-NK_1=(1-\lambda^L)f. \qquad (3.12)$$

From (3.12), we have

$$K_1=\frac{1-(1-\lambda^L)f}{N}=\frac{1-(1-\lambda^L)f}{\lambda^m b}. \qquad (3.13)$$

From (3.13), we easily obtain the necessary and sufficient condition for $K_1\in \boldsymbol{R}[\lambda]$:

$$\Omega[\lambda^m b]\subset \Omega[1-(1-\lambda^L)f]. \qquad (3.14)$$

Based on (3.14), we can determine the lowest polynomial f^*

$$f^*=f_0^*+f_1^*\lambda+...+f_{m+l-1}^*\lambda^{m+l-1} \qquad (3.15)$$

exactly. Here, for simplicity, we assume that $b(\lambda)=0$ has only simple roots and let $\xi_1,\xi_2,...,\xi_l$ denote these roots. The result is summarized as follows:

Theorem 1: The parameter K_1 which yields low–ripple, repetitive deadbeat control with a minimum settling step is given by

$$K_1^*=\frac{1-(1-\lambda^L)f^*}{N}, \qquad (3.16)$$

where f^* is the polynomial given in (3.15) and its coefficients are determined by the following algorithm:

1) $f_0^*, f_1^*,..., f_{m-1}^*$ are determined by the m–multiple original zero:

If $L>m-1$:

$$f_i^*=\begin{cases}1 & i=0;\\ 0 & i=1,2,...,m-1.\end{cases} \qquad (3.17)$$

If $L\le m-1$: let η be an positive integer which satisfys $\eta\le\frac{m-1}{L}<\eta+1$. Then

$$f_i^*=\begin{cases}1 & i=kL\ (k=0,1,2,...,\eta);\\ 0 & i\ne kL\ (k=0,1,2,...,\eta)\ \&\ i\le m\text{-}1.\end{cases} \qquad (3.18)$$

2) $f_m^*, f_{m+1}^*,..., f_{m+l-1}^*$ are determined by the l–simple zeros $\xi_1,\xi_2,...,\xi_l$:

$$\begin{bmatrix} f_m^* \\ f_{m+1}^* \\ \vdots \\ f_{m+l-1}^* \end{bmatrix}=\begin{bmatrix} \xi_1^{\,m} & \xi_1^{\,m+1} & \cdots & \xi_1^{\,m+l-1} \\ \xi_2^{\,m} & \xi_2^{\,m+1} & \cdots & \xi_2^{\,m+l-1} \\ \vdots & \vdots & \vdots & \vdots \\ \xi_l^{m} & \xi_l^{m+1} & \cdots & \xi_l^{m+l-1} \end{bmatrix}^{-1}\begin{bmatrix} g(\xi_1) \\ g(\xi_2) \\ \vdots \\ g(\xi_l) \end{bmatrix}, \qquad (3.19)$$

where $g(\lambda):=\frac{1}{1-\lambda^L}-\sum_{i=0}^{m-1}f_i^*\lambda^i$. (3.20)

However, as is well known, minimum–settling–step deadbeat control may have a violent transient response. To avoid this we can use the rest of the freedom of parameter K_1 to optimize the transient response.

From (3.10), (3.13) and (3.16), it is clear that all K_1 which yield low–ripple repetitive deadbeat control can be parametrized as

$$K_1=K_1^*+(1-\lambda^L)\overline{K}_1; \quad \overline{K}_1\in \boldsymbol{R}[\lambda], \qquad (3.21)$$

where $\overline{K}_1$ can be any polynomial. So we can choose an appropriate polynomial $\overline{K}_1\ne 0$ to optimize the transient response.

Let

$$\overline{K}_1=\bar{k}_0+\bar{k}_1\lambda+\cdots+\bar{k}_q\lambda^q=\sum_{i=0}^{q}\bar{k}_i\lambda^i. \qquad (3.22)$$

Then we have

$$\tilde{e}=(1-NK_1)\frac{\bar{r}}{1-\lambda^L}=f^*\bar{r}-N\bar{r}\overline{K}_1$$
$$:=\tilde{e}^*-\phi\overline{K}_1=\sum_{i=0}^{L+m+l+q-1}\tilde{e}_i\lambda^i, \qquad (3.23)$$

where

$$\tilde{e}^*:=f^*\bar{r}=\sum_{i=0}^{L+m+l\text{-}2}\tilde{e}_i^*\lambda^i \qquad (3.24a)$$

$$\phi:=N\bar{r}=\sum_{i=0}^{L+m+l\text{-}1}\phi_i\lambda^i. \qquad (3.24b)$$

If we assume

$$\Delta\tilde{u} := (1-\lambda^L)\tilde{u}\,, \tag{3.25}$$

then

$$\begin{aligned}\Delta\tilde{u}(\lambda) &= (1-\lambda^L)G_{\tilde{u}\tilde{r}}\tilde{r} = DK_1\bar{r} \\ &= DK_1^*\bar{r} + (1-\lambda^L)D\bar{r}\bar{K}_1 \\ &:= \Delta\tilde{u}^* - \gamma\bar{K}_1 = \sum_{i=0}^{2L+n+q-1}\Delta\tilde{u}_i\lambda^i,\end{aligned} \tag{3.26}$$

where

$$\Delta\tilde{u}^* := DK_1^*\bar{r} = \sum_{i=0}^{2L+n-2}\Delta\tilde{u}_i^*\lambda^i \tag{3.27a}$$

$$\gamma := (1-\lambda^L)D\bar{r} = \sum_{i=0}^{2L+n-1}\gamma_i\lambda^i. \tag{3.27b}$$

Generalizing the transient response performance index given in [11] and [12], we define it to be

$$J_1 = \sum_{i=0}^{\mu+L+q-1}\{|\tilde{e}_i|^2 + \rho^2|\Delta\tilde{u}_i|^2\}, \tag{3.28}$$

where μ=max$\{m+l, L+n\}$ and ρ is a weighting coefficient; and let

$$\bar{\boldsymbol{K}}_1 := [\bar{k}_0\ \bar{k}_1\ \cdots\ \bar{k}_q]^{\mathrm{T}} \in \boldsymbol{R}^{q+1} \tag{3.29a}$$

$$\tilde{\boldsymbol{e}} := [\tilde{e}_0\ \tilde{e}_1\ \cdots\ \tilde{e}_{L+m+l+q-1}]^{\mathrm{T}} \in \boldsymbol{R}^{L+m+l+q} \tag{3.29b}$$

$$\tilde{\boldsymbol{e}}^* := [\tilde{e}_0^*\ \tilde{e}_1^*\ \cdots\ \tilde{e}_{L+m+l-2}^*\ 0\ \cdots\ 0]^{\mathrm{T}} \in \boldsymbol{R}^{L+m+l+q} \tag{3.29c}$$

$$\Delta\tilde{\boldsymbol{u}} := [\Delta\tilde{u}_0\ \Delta\tilde{u}_1\ \cdots\ \Delta\tilde{u}_{2L+n+q-1}]^{\mathrm{T}} \in \boldsymbol{R}^{2L+n+q} \tag{3.29d}$$

$$\Delta\tilde{\boldsymbol{u}}^* := [\Delta\tilde{u}_0^*\ \Delta\tilde{u}_1^*\ \cdots\ \Delta\tilde{u}_{2L+n-2}^*\ 0\ \cdots\ 0]^{\mathrm{T}} \in \boldsymbol{R}^{2L+n+q}. \tag{3.29e}$$

Then the performance index can be expressed as

$$J_1 = \|\tilde{\boldsymbol{e}}\|_2^2 + \rho^2\|\Delta\tilde{\boldsymbol{u}}\|_2^2 = \left\| -\begin{bmatrix}\boldsymbol{\Phi} \\ -\rho\boldsymbol{\Gamma}\end{bmatrix}\bar{\boldsymbol{K}}_1 + \begin{bmatrix}\tilde{\boldsymbol{e}}^* \\ \rho\Delta\tilde{\boldsymbol{u}}^*\end{bmatrix}\right\|_2^2, \tag{3.30}$$

where

$$\tilde{\boldsymbol{e}} = \tilde{\boldsymbol{e}}^* - \boldsymbol{\Phi}\bar{\boldsymbol{K}}_1 \tag{3.31a}$$

$$\Delta\tilde{\boldsymbol{u}} = \Delta\tilde{\boldsymbol{u}}^* - \boldsymbol{\Gamma}\bar{\boldsymbol{K}}_1 \tag{3.31b}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} \phi_0 & 0 & \cdots & 0 \\ \phi_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \phi_{L+m+l-1} & & \ddots & \phi_0 \\ 0 & \ddots & & \phi_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \phi_{L+m+l-1} \end{bmatrix} \in \boldsymbol{R}^{(L+m+l+q)\times(q+1)} \tag{3.31c}$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} \gamma_0 & 0 & \cdots & 0 \\ \gamma_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \gamma_{2L+n-1} & & \ddots & \gamma_0 \\ 0 & \ddots & & \gamma_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \gamma_{2L+n-1} \end{bmatrix} \in \boldsymbol{R}^{(2L+n+q)\times(q+1)}. \tag{3.31d}$$

The solution of $\bar{K}_1$ is given in the following theorem.

Theorem 2: The coefficients of $\bar{K}_1$ in (3.29a) which minmize the transient response performance index (3.28) are given by

$$\bar{\boldsymbol{K}}_1 = F_1^{-1}F_2, \tag{3.32}$$

where

$$F_1 = \begin{bmatrix}\boldsymbol{\Phi}^{\mathrm{T}} & -\rho\boldsymbol{\Gamma}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Phi} \\ -\rho\boldsymbol{\Gamma}\end{bmatrix} \tag{3.33a}$$

$$F_2 = \begin{bmatrix}\boldsymbol{\Phi}^{\mathrm{T}} & -\rho\boldsymbol{\Gamma}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{e}}^* \\ \rho\Delta\tilde{\boldsymbol{u}}^*\end{bmatrix}; \tag{3.33b}$$

and the minimum of J_1 is given by

$$\min J_1 = J_{1M} - F_2^{\mathrm{T}}F_1^{-1}F_2, \tag{3.34}$$

where

$$\begin{aligned}J_{1M} &= \|\tilde{\boldsymbol{e}}^*\|_2^2 + \rho^2\|\Delta\tilde{\boldsymbol{u}}^*\|_2^2 \\ &= \sum_{i=0}^{L+m+l-2}\tilde{e}_i^{*2} + \rho^2\sum_{i=0}^{2L+n-2}\Delta\tilde{u}_i^{*2}.\end{aligned} \tag{3.35}$$

Design of Parameter K_2. As we saw in Section Ⅱ, in the "position domain", when the scanning speed fluctuates periodically, so does the plant. If we denote such a plant as $\hat{P}$, then the nominal and actual input–output transfer functions of the system are given by

$$G_{\tilde{y}\tilde{r}} = \frac{PC_1}{1+PC_2} \tag{3.36a}$$

$$\hat{G}_{\tilde{y}\tilde{r}} = \frac{\hat{P}C_1}{1+\hat{P}C_2}. \tag{3.36b}$$

The change in the input–output transfer functions of the system due to fluctuations of the scanning speed can be characterized as

$$\frac{\hat{G}_{\tilde{y}\tilde{r}} - G_{\tilde{y}\tilde{r}}}{\hat{G}_{\tilde{y}\tilde{r}}} = \frac{1}{1+PC_2}\frac{\hat{P}-P}{\hat{P}} = S\frac{\hat{P}-P}{\hat{P}}, \tag{3.37}$$

where

$$S := \frac{1}{1+PC_2} \tag{3.38}$$

is the sensitivity function of the system. From (3.37), it is clear that the system will be robust if the weighted sensitivity function is made as small as possible. For this reason, we define the robust index as

$$J_2 = \|WS\|_\infty = \left\|\frac{W}{1+PC_2}\right\|_\infty. \tag{3.39}$$

Considering that in our problem the scanning speed fluctuates periodically, we should choose the weighting function to be

$$W = \frac{\bar{W}}{1-\lambda^L}, \tag{3.40}$$

and thus (3.39) becomes

$$J_2 = \left\|\frac{\bar{W}}{1-\lambda^L}\frac{1}{1+PC_2}\right\|_\infty = \|\bar{W}D(Y'-K_2'N)\|_\infty. \tag{3.41}$$

Then the parameter $K_2' \in \boldsymbol{RH}_\infty$ can be determined by choosing it to yield $\inf_{K_2'\in\boldsymbol{RH}_\infty} J_2$. The solution is given in [13], [14], [15].

Ⅳ EXPERIMENT

The experimental system is shown in Fig. 3. It consists of a pen, a disk, a one–axle table, two DC motors, a computer with a 68000–series CPU and the relevant interface hardware. One of the DC motors is used to turn the disk for the purpose of generating a standard position. The other is used to drive the table with the pen connected to it. By controlling the position of the table, we want to

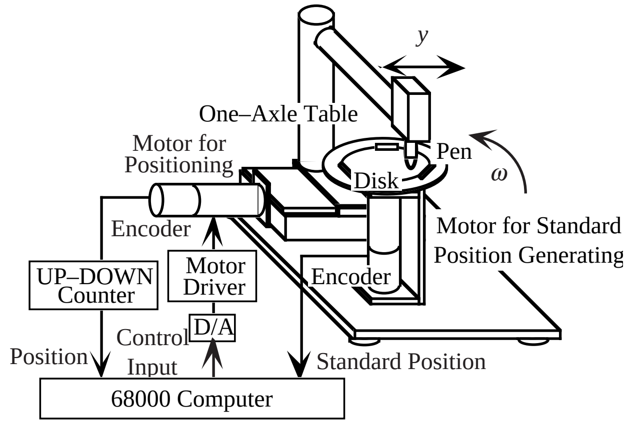

Fig. 3 Experemental set–up.

draw a flower pattern on the disk.

The plant model in the time domain is

$$P(s) = \frac{0.6547}{s(0.09719s+1)}. \tag{4.1}$$

Assuming that the standard scanning speed is

$$\omega_0 = 5.236(rad/s), \tag{4.2}$$

the transformation introduced in Ⅱ yield a plant model in the "position domain". Then, we sample it at intervals of

$$\Delta\theta = 0.1257(rad) \tag{4.3}$$

to obtain the pulse transfer function of the nominal plant:

$$P = \frac{\lambda(\beta_1\lambda + \beta_0)}{(1-\lambda)(\lambda-\alpha)} \tag{4.4a}$$

$$\alpha = 1.280;\ \beta_0 = -0.002291;\ \beta_1 = -0.002110. \tag{4.4b}$$

Let the input waveform be

$$\tilde{r}(\theta) = 20\sin 2\theta(mm). \tag{4.5}$$

Then we can design a TDF repetitive controller for (4.4) using the approach developed in Ⅲ.

After carrying out the coprime factorization (3.3), we use Theorem 1 to calculate the parameter K_1^*. Then letting

$$q = 100 \tag{4.6}$$

$$\rho = 1, \tag{4.7}$$

we use Theorem 2 to calculate the parameter $\overline{K}_1$. Combining these two parameters according to (3.21), we get the resultant K_1. For the calculation of K_2', we take

$$\overline{W} = \frac{1}{1-\lambda} \tag{4.8}$$

and solve the problem $\inf_{K_2' \in \boldsymbol{RH}_\infty} J_2$ in (3.41). Finally, substituting K_1 and K_2' into (3.8), we obtain the TDF repetitive controller.

The experimental results are shown in Figs. 4–5. In Fig. 4, the scanning speed is constant. And in Fig. 5, it fluctuates periodically with a period of 2π (rad). The experimental results show that the present method provides high positioning precision even when the scannig speed fluctuates periodically.

Ⅴ CONCLUSIONS

This note describes a design method for solving the positioning control problem in the case where a reference input is scanned at a position–dependent periodic speed. A new concept called the "position domain" is introduced, and a transformation from the time domain to the "position domain" is presented. For the purpose of optimizing the input–output response and ensuring robustness, a TDF repetitive control configuration is employed and a design method for this system is presented. The validity of the present method has been demonstrated by experiments.

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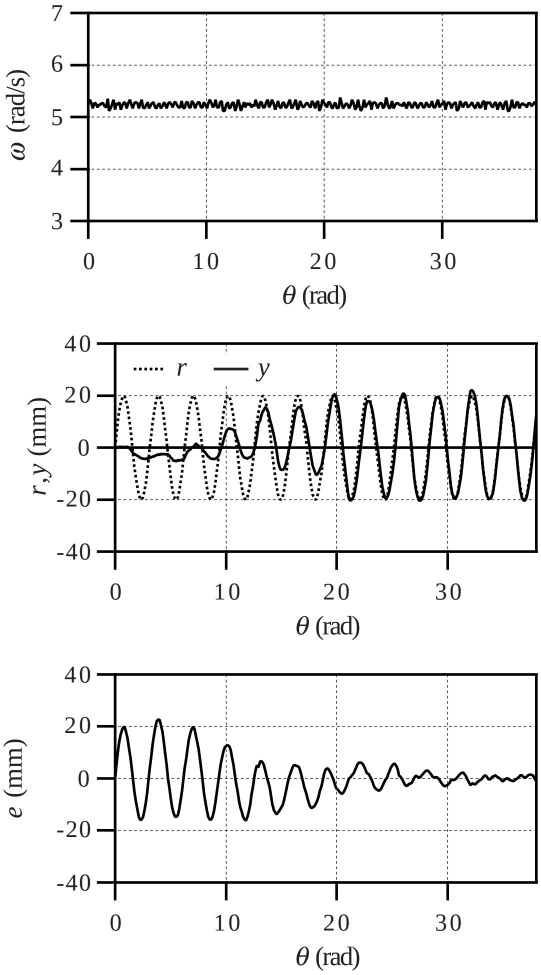

Fig. 4 Experimental results for a constant scanning speed.

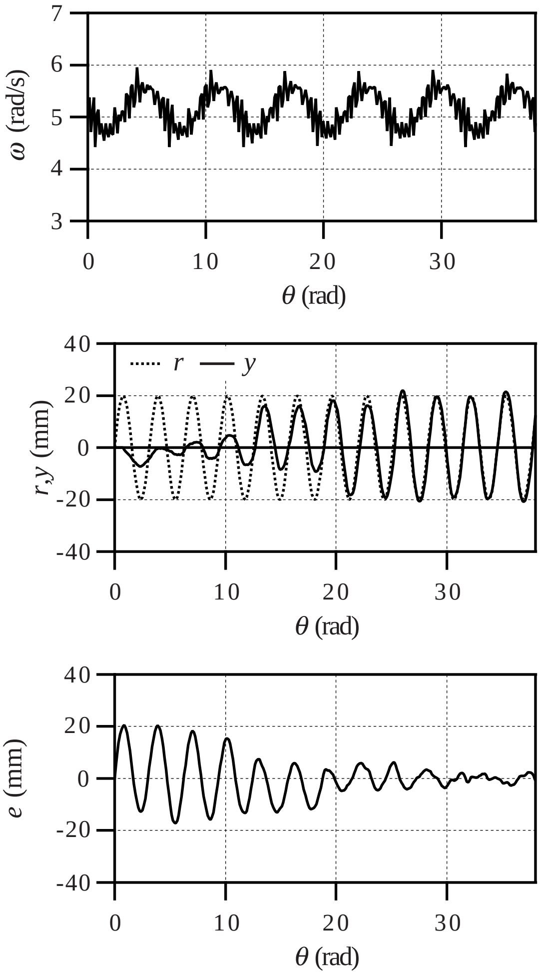

Fig. 5 Experimental results for a periodically fluctuating scanning speed.