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A Design Methodology for Robust Two–Degree–of–Freedom Digital Repetitive Control

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Abstract: This paper presents a design method for digital repetitive control systems for a plant with structured uncertainties. A two-degree-of-freedom (TDF) control system configuration has been employed in order to achieve the desired feedback and input-output performance independently. The lowest-order feedback repetitive controller is designed to guarantee the robust stability of the closed-loop system and provide the desired closed-loop performance. A feedforward repetitive deadbeat controller has been designed to obtain the desired transient response.

Keywords: Repetitive Control, Two-Degree-of-Freedom, Sampled-Data Control, Discrete-Time H^∞ control, Deadbeat Control.

1. INTRODUCTION

Repetitive control is a very useful strategy for tracking periodic signals and/or eliminating periodic disturbances. However, since a repetitive controller usually has a large time delay, the design of a low-order robust repetitive controller is difficult, and system design has mainly focused on the nominal plant (Hara *et al*, 1988; Tomizuka *et al.*, 1989). However, there has been significant progress in robust control theory recently, and some interesting results on robust repetitive control have been obtained by Hoshi *et al.* (1993), Ishibashi *et al.* (1994), Hara *et al.* (1994b) and Shaw and Srinvasan (1993).

Motivated by the recent development of the sampled-data H^∞ control theorem by such researchers as Bamieh and Pearson (1992), Fujioka and Hara (1993), Hara *et al.* (1994a), Hayakawa *et al.* (1992), and Kabamba and Hara (1993), and of the static output feedback H^∞ control theorem (de Souza and Xie, 1992), we present a design method for digital repetitive control systems which robustly stabilizes an uncertain plant. The present design method features the lowest order of the repetitive feedback controller.

Throughout this paper, λ denotes a delay operator, and $\boldsymbol{R}[\lambda]$ denotes a ring of polynomials in λ.

2. PROBLEM FORMULATION

Let us consider the partial-static-state-feedback repetitive control system shown in Fig. 1, where the repetitive controller C_R,

$$\begin{cases} C_R = \dfrac{f_L\lambda + f_{L-1}\lambda^2 + \cdots + f_1\lambda^L}{1-\lambda^L} := \dfrac{F\left[\lambda^L \;\; \lambda^{L-1} \;\; \cdots \;\; \lambda\right]^T}{1-\lambda^L} \\ F := \left[f_1 \;\; \cdots \;\; f_{L-1} \;\; f_L\right], \end{cases} \tag{1}$$

can have a controllable canonical structure

$$C_R = \left[\begin{array}{c|c} \begin{bmatrix} 0 & I_{L-1} \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \hline F & 0 \end{array}\right], \tag{2}$$

or an observable canonical structure

$$C_R = \left[\begin{array}{c|c} \begin{bmatrix} 0 & 1 \\ I_{L-1} & 0 \end{bmatrix} & F \\ \hline \begin{bmatrix} 0 & 1 \end{bmatrix} & 0 \end{array}\right]. \tag{3}$$

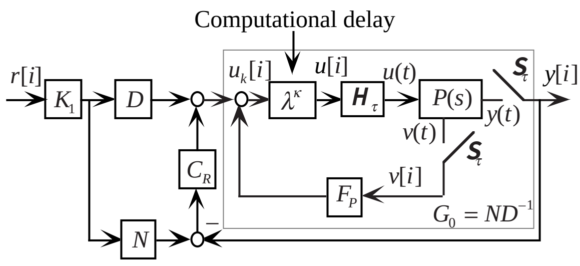

Fig. 1. Configuration of TDF repetitive control system.

In this paper, we use the controllable canonical form for analysis; and we use the observable canonical form for implementation because it automatically incorporates a one-step computational delay of the repetitive controller. The repetitive control system in Fig. 1 contains a κ-step computational delay.

Consider the structurally uncertain plant $P(s)$ (Asai and Hara, 1992)

$$\begin{cases} (E+\Phi\Gamma(t)\Psi_E)\dot{x}_P(t) \\ \quad =(A+\Phi\Gamma(t)\Psi_A)x_P(t)+(B+\Phi\Gamma(t)\Psi_B)u(t) \\ y(t)=Cx_P(t) \\ v(t)=Hx_P(t) \\ \Gamma^T(t)\Gamma(t)\le I; \quad E:\text{non-singular,} \end{cases} \tag{4}$$

where $x(t)\in \boldsymbol{R}^n$ is the state, $u(t)\in \boldsymbol{R}$ is the control input, $y(t)\in \boldsymbol{R}$ is the output and $v(t)\in \boldsymbol{R}^p$ is the available partial state.

The nominal plant $P_0(s)$ corresponding to $\Gamma(t)=0$ is

$$\begin{cases} E\dot{x}_P(t)=Ax_P(t)+Bu(t) \\ y(t)=Cx_P(t) \\ v(t)=Hx_P(t). \end{cases} \tag{5}$$

To make this problem solvable, we need two assumptions.

Assumption 1: $\left(E^{-1}A \quad E^{-1}B\right)$ is stabilizable.
Assumption 2: The sampling period τ is chosen such that $(e^{E^{-1}A\tau} \quad \int_0^\tau e^{E^{-1}A(\tau-t)}E^{-1}Bdt)$ is stabilizable.

Here, we represent the feedback controller in Fig. 1 by

$$\boldsymbol{K}:\begin{cases} x_k[i+1]=A_kx_k[i]+B_kS_\tau\left[y(t) \quad v(t)\right]^T \\ u_k[i]=C_kx_k[i]+D_kS_\tau\left[y(t) \quad v(t)\right]^T \end{cases} \tag{6}$$

and the computational delay by

$$\boldsymbol{C_D}:\begin{cases} x_d[i+1]=A_dx_d[i]+B_du_k[i] \\ u(t)=\mathrm{H}_\tau C_dx_d[i], \end{cases} \tag{7}$$

where

$$\begin{cases} \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right]=\left[\begin{array}{c|c} \begin{bmatrix} 0 & I_{L-1} \\ 1 & 0 \end{bmatrix} & \left[\begin{bmatrix} 0_{(L-1)\times 1} \\ -1 \end{bmatrix} \quad 0\right] \\ \hline F & 0 \quad F_P \end{array}\right] \\ \left[\begin{array}{c|c} A_d & B_d \\ \hline C_d & 0 \end{array}\right]=\left[\begin{array}{c|c} \begin{bmatrix} 0 & I_{\kappa-1} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0_{(\kappa-1)\times 1} \\ 1 \end{bmatrix} \\ \hline \begin{bmatrix} 1 & 0_{1\times(\kappa-1)} \end{bmatrix} & 0 \end{array}\right]. \end{cases} \tag{8}$$

Then the closed-loop system with zero input is represented by

$$\Sigma:\begin{cases} (E+\Phi\Gamma(t)\Psi_E)\dot{x}_P(t) \\ \quad =(A+\Phi\Gamma(t)\Psi_A)x_P(t)+(B+\Phi\Gamma(t)\Psi_B)H_\tau C_dx_d[i] \\ x_k[i+1]=B_kS_\tau\left[C \quad H\right]^T x_P(t)+A_kx_k[i] \\ x_d[i+1]=B_dD_kS_\tau\left[C \quad H\right]^T x_P(t)+B_dC_kx_k[i]+A_dx_d[i] \\ y(t)=Cx_P(t). \end{cases} \tag{9}$$

The robust stability of this repetitive control system can be defined as follows (Hara *et al*., 1994b):

Definition The repetitive control system in Fig. 1 is said to be robustly stable if the closed loop system Σ is asymptotically stable.

Now the design problem for repetitive control systems can be stated as:

i) Design a static feedback gain $\left[F_P \quad F\right]$ which robustly stabilizes the system in Fig. 1.
ii) Design a feedforward controller K_1 which yields the desired transient input-output response.

3. DESIGN OF FEEDBACK CONTROLLER

Redrawing Fig. 1 as Fig. 2 with the input $r[i]=0$, we obtain the condition for robust stability by applying the small gain theorem (Sivashankar and Khargonekar, 1993):

Lemma 3.1 The repetitive control system in Fig. 1 is robustly stable if

$$\|G\|_\infty<1, \tag{10}$$

where

$$\|G\|_\infty := \sup_{w\in L_2}\frac{\|Gw\|_2}{\|w\|_2}=\sup_{w\in L_2}\frac{\|z\|_2}{\|w\|_2}. \tag{11}$$

To guarantee the robust stability of the system and obtain the desired closed-loop performance, we extend the controlled output and include the weighted control input and the states of the plant, controller and computational delay in it. Now the design problem for the feedback controller can be formulated as (see Fig. 3):

Find a static feedback gain $\left[F_P \quad F\right]$ which internally stabilizes $\boldsymbol{P}_s$ and satisfys

$$\|G_{zw}\|_\infty<1, \tag{12}$$

where P_s is given by

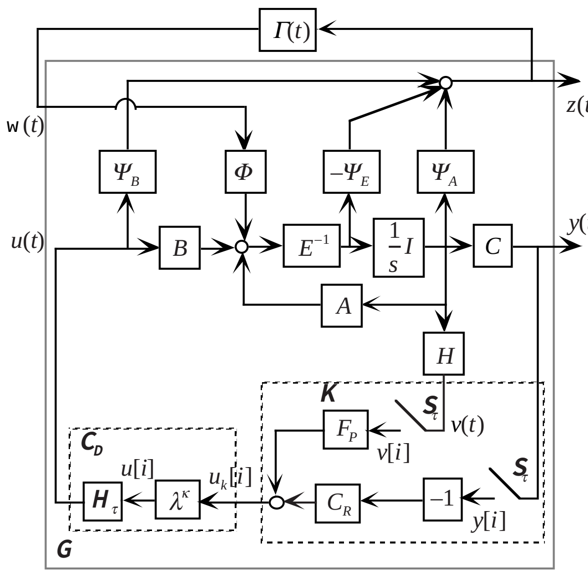

Fig. 2. Robust stability.

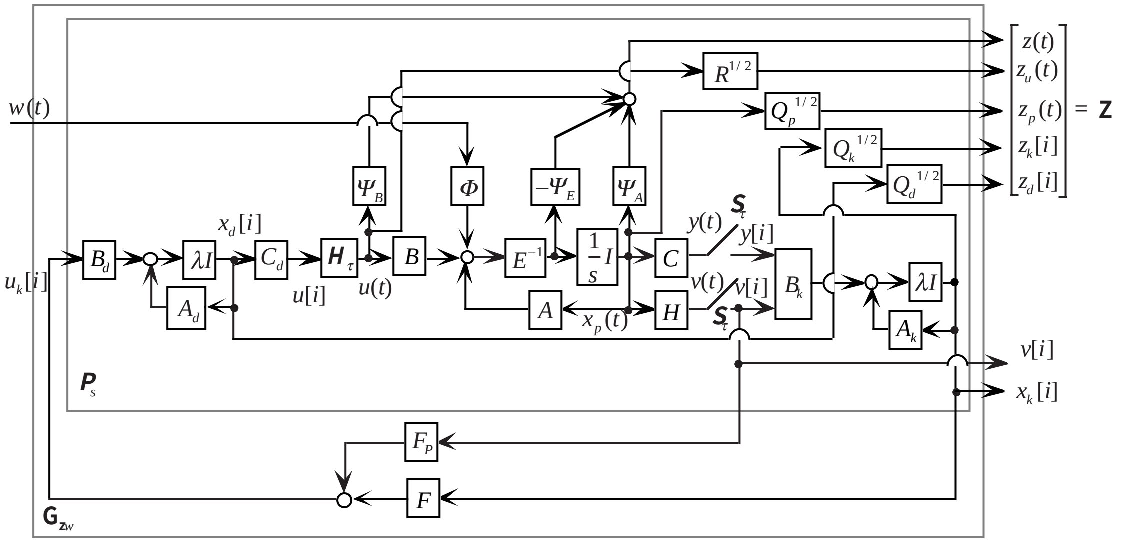

Fig. 3. Formulation of feedback control problem.

$$P_s = \left[\begin{array}{c|c:c} \begin{bmatrix} E^{-1}A & 0 & E^{-1}BH_\tau C_d \\ B_k S_\tau \begin{bmatrix} C \\ H \end{bmatrix} & A_k & 0 \\ 0 & 0 & A_d \end{bmatrix} & \begin{bmatrix} E^{-1}\Phi \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ B_d \end{bmatrix} \\ \hline \begin{bmatrix} \Psi_A - \Psi_E E^{-1}A & 0 & (\Psi_B - \Psi_E E^{-1}B)H_\tau C_d \\ 0 & 0 & R^{1/2}H_\tau C_d \\ Q_p^{1/2} & 0 & 0 \\ 0 & Q_k^{1/2} & 0 \\ 0 & 0 & Q_d^{1/2} \end{bmatrix} & \begin{bmatrix} -\Psi_E E^{-1}\Phi \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} & 0 \\ \hdashline \begin{bmatrix} S_\tau H & 0 & 0 \\ 0 & I & 0 \end{bmatrix} & 0 & 0 \end{array}\right]. \tag{13}$$

Applying Theorem 2 of Fujioka and Hara (1993) to our case, we have the following theorem.

Theorem 3.1 The following are equivalent:

i) The system of Fig. 3 is internally stable and $\|G_{zw}\|_\infty < 1$;

ii) The system of Fig. 4 is internally stable and $\|G_e(\lambda)\|_\infty < 1$.

The discrete equivalent plant $P_e(\lambda)$ in Fig. 4 is obtained by the following procedure. First, decompose the plant P_s into two sub-systems, P_C and $P_D(\lambda)$, as shown in Fig. 5. Secondly, lift the sub-system P_C and reduce it to a finite-dimensional discrete system. Finally, combining this system with $P_D(\lambda)$ yields the discrete equivalent plant $P_e(\lambda)$.(A special case of this problem ($\Psi_E = 0$) can be calculated using the MATLAB μ-toolbox (Balas, *et al*, 1993).) Let the equivalent general discrete-time plant be

$$P_e = \left[\begin{array}{c|c:c} A_e & B_{e1} & B_{e2} \\ \hline C_{e1} & D_{e11} & D_{e12} \\ \hdashline C_{e2} & 0 & 0 \end{array}\right], \tag{14}$$

Then the design problem is equivalent to the following static-output-feedback discrete-time H^∞ control problem:

Find a static output feedback gain $[F_P \quad F]$ which internally stabilizes $P_e(\lambda)$ and satisfies (see Fig. 4)

$$\|G_e(\lambda)\|_\infty < 1. \tag{15}$$

The resulting $[F_P \quad F]$ can be calculated by the following algorithm (de Souza and Xie, 1992):

Algorithm 3.1 Set

$$\begin{cases} \hat{B} = [B_{e1} \quad B_{e2}]; \quad \hat{D} = [D_{e11} \quad D_{e12}] \\ S = [0 \quad I]; \quad \hat{R} = \hat{D}^T\hat{D} - [I \quad 0]^T[I \quad 0]. \\ \Omega = I - C_{e2}{}^T(C_{e2}C_{e2}{}^T)^{-1}C_{e2} \end{cases} \tag{16}$$

Step 1: Set the iteration index i=1 and Q_1=0.
Step 2: Solve the following discrete algebraic Riccati equation:

$$\begin{aligned} &A_e{}^T P_i A_e - P_i - (A_e{}^T P_i \hat{B} + C_{e1}{}^T \hat{D}) \\ &*(\hat{B}^T P_i \hat{B} + \hat{R})^{-1}(\hat{B}^T P_i A_e + \hat{D}^T C_{e1}) + Q_i + C_{e1}{}^T C_{e1} = 0 \end{aligned} \tag{17}$$

for P_i. If $P_i \geq 0$, then go to step 3; otherwise, no feasible solution was found.

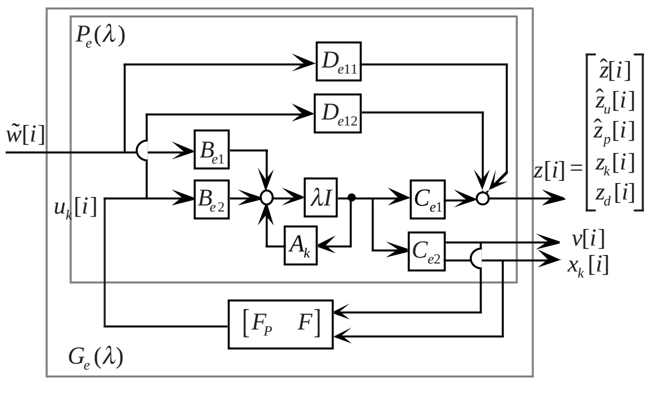

Fig. 4. Equivalent discrete-time system.

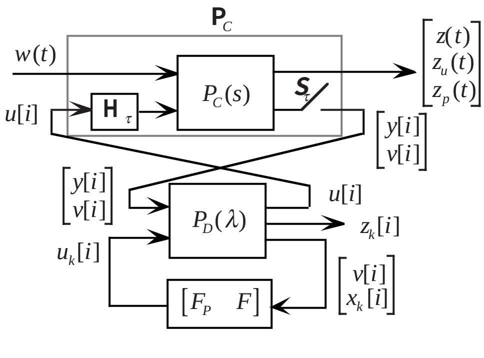

Fig. 5. System decomposition.

Step 3: If $\|P_i - P_{i-1}\|/\|P_i\| < \varepsilon$, where ε is a small positive real number, proceed to the next step; otherwise, let i=i+1 and set

$$\begin{cases} \Pi_{i-1} = (\hat{B}^T P_{i-1}\hat{B} + \hat{R})^{-1} \\ \alpha(P_{i-1}) = S\Pi_{i-1}(\hat{B}^T P_i A_e + \hat{D}^T C_{e1}); \quad \beta(P_{i-1}) = S\Pi_{i-1}S^T \\ L_i = \beta^{-1}(P_{i-1})\alpha(P_{i-1})\Omega; \quad Q_i = L_i{}^T S\Pi_{i-1}L_i \end{cases}. \tag{18}$$

If $Q_i < 0$, then set $Q_i = Q_{i-1}$. Go back to step 2.

Step 4: If $V_i = I - (B_{e1}{}^T P_i B_{e1} + D_{e11}{}^T D_{e11}) > 0$, then go step 5; otherwise, no feasible solution was found.
Step 5: Calculate the feedback gain by

$$\begin{bmatrix} F_P & F \end{bmatrix} = -\alpha(P_i)C_{e2}{}^T(C_{e2}C_{e2}{}^T)^{-1}. \tag{19}$$

4. DESIGN OF FEEDFORWARD CONTROLLER

Assume an L-periodic signal to be

$$r(\lambda) = \frac{\bar{r}(\lambda)}{1-\lambda^L} = \frac{r_0 + r_1\lambda + r_2\lambda^2 + ... + r_{L-1}\lambda^{L-1}}{1-\lambda^L}. \tag{20}$$

Let $G_P(\lambda)$ denote the pulse transfer function of the nominal plant with a κ-step computational delay, and $G_0(\lambda)$ denote the pulse transfer function of $G_P(\lambda)$ with partial state feedback F_P. Also, let their coprime factorizations be

$$\begin{cases} G_P = \dfrac{N}{D_P} = \dfrac{\lambda^m b}{\alpha} = \dfrac{\lambda^m(b_0 + b_1\lambda + ... + b_\ell\lambda^\ell)}{\alpha_0 + \alpha_1\lambda + ... + \alpha_n\lambda^n} \\ N = \lambda^m b; \quad D_P = \alpha \end{cases}. \tag{21}$$

and

$$\begin{cases} G_0 = \dfrac{N}{D} = \dfrac{\lambda^m b}{a} = \dfrac{\lambda^m(b_0 + b_1\lambda + ... + b_\ell\lambda^\ell)}{a_0 + a_1\lambda + ... + a_n\lambda^n} \\ N = \lambda^m b; \quad D = a \end{cases}, \tag{22}$$

For simplicity, we assume that no cancellation between the zeros and poles of the plant is brought about by the introduction of the partial state feedback F_P.

We can design a low-ripple deadbeat feedforward controller, K_1, in accordance with G_0 and G_P. The term "low–ripple deadbeat response" means that the following conditions are satisfied:

(i) *Deadbeat condition:* The tracking error between the reference input and the output is a finite polynomial, i.e. $e := r - y = \sum_{i=0}^{\mu} e_i\lambda^i$, where μ is a finite positive integer.

(ii) *Low–ripple condition:* The transfer function from the reference input to the control input is a finite polynomial, i.e. $G_{u_k r} \in \boldsymbol{R}[\lambda]$.

Some of the details are given in She (1993). Here the main results are just summarized in the following three theorems.

Theorem 4.1 The parameter K_1 which yields low–ripple, repetitive deadbeat control with a minimum settling time is given by

$$K_1{}^* = \frac{1-(1-\lambda^L)f^*}{N}, \tag{23}$$

where f^* is the polynomial

$$f^* = f_0^* + f_1^*\lambda + ... + f_{m+\ell-1}^*\lambda^{m+\ell-1}, \tag{24}$$

and its coefficients are determined by the following algorithm (For simplicity, we assume that $b(\lambda) = 0$ has only simple roots and let $\xi_1, \xi_2, ..., \xi_l$ denote these roots.):

1) $f_0^*, f_1^*, ..., f_{m-1}^*$ are determined by the m–multiple original zero of G_0:
If $L > m-1$,

$$f_i^* = \begin{cases} 1 & i = 0; \\ 0 & i = 1, 2, ..., m-1. \end{cases} \tag{25}$$

If $L \le m-1$, let η be a positive integer which satisfies $\eta \le \dfrac{m-1}{L} < \eta + 1$. Then

$$f_i^* = \begin{cases} 1 & i = kL \ (k = 0, 1, 2, ..., \eta); \\ 0 & i \ne kL \ (k = 0, 1, 2, ..., \eta) \ \& \ i \le m\text{-}1. \end{cases} \tag{26}$$

2) $f_m^*, f_{m+1}^*, ..., f_{m+\ell-1}^*$ are determined by the ℓ–simple zeros, $\xi_1, \xi_2, ..., \xi_\ell$, of G_0:

$$\begin{bmatrix} f_m^* \\ f_{m+1}^* \\ \vdots \\ f_{m+\ell-1}^* \end{bmatrix} = \begin{bmatrix} 1 & \xi_1 & \cdots & \xi_1^{\ell-1} \\ 1 & \xi_2 & \cdots & \xi_2^{\ell-1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \xi_\ell & \cdots & \xi_\ell^{\ell-1} \end{bmatrix}^{-1} \begin{bmatrix} g(\xi_1) \\ g(\xi_2) \\ \vdots \\ g(\xi_\ell) \end{bmatrix}, \tag{27}$$

where

$$g(\lambda) := \frac{1}{\lambda^m(1-\lambda^L)} - \sum_{i=0}^{m-1} f_i^*\lambda^{i-m}. \tag{28}$$

Theorem 4.2 All K_1 that yield low–ripple repetitive deadbeat control can be parameterized as

$$K_1 = K_1{}^* + (1-\lambda^L)\bar{K}_1; \ \bar{K}_1 \in \boldsymbol{R}[\lambda], \tag{29}$$

where $\bar{K}_1$ is any polynomial.

To optimize the transient response, we choose an appropriate non-zero polynomial

$$\bar{K}_1 = \bar{k}_0 + \bar{k}_1\lambda + \cdots + \bar{k}_q\lambda^q = \sum_{i=0}^{q}\bar{k}_i\lambda^i \ne 0, \tag{30}$$

and use it to optimize the following performance:

$$J = \sum_{i=0}^{L+m+\ell+q-1} \{\|e[i]\|^2 + \rho^2 |\Delta u[i]\|^2\}, \tag{31}$$

where

$$\begin{cases} e[i] := r[i] - y[i] \\ \Delta u_k[i] := u_k[i] - \bar{u}_k[i]; \ \ \bar{u}[i]_k : u_k[i] \text{ in steady state,} \end{cases} \tag{32}$$

and ρ is a weighting coefficient. The resulting $\bar{K}_1$ is given by the following theorem.

Theorem 4.3 The coefficients of $\bar{K}_1$,

$$\bar{\boldsymbol{K}}_1 := [\bar{k}_0 \ \bar{k}_1 \ \cdots \ \bar{k}_q]^T \in \boldsymbol{R}^{q+1}, \tag{33}$$

that minmize the transient response performance index (31) are given by

$$\bar{\boldsymbol{K}}_1 = F_1^{-1} F_2, \tag{34}$$

where

$$\begin{cases} F_1 = \begin{bmatrix} \boldsymbol{\Phi}^T & -\rho \boldsymbol{\Gamma}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{\Phi} \\ -\rho \boldsymbol{\Gamma} \end{bmatrix} \\ F_2 = \begin{bmatrix} \boldsymbol{\Phi}^T & -\rho \boldsymbol{\Gamma}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{e}^* \\ \rho \Delta \boldsymbol{u}^* \end{bmatrix} \end{cases}, \tag{35}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} \phi_0 & 0 & \cdots & 0 \\ \phi_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \phi_{L+m+\ell-1} & & \ddots & \phi_0 \\ 0 & \ddots & & \phi_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \phi_{L+m+\ell-1} \end{bmatrix} \in \boldsymbol{R}^{(L+m+\ell+q)\times(q+1)}, \tag{36}$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} \gamma_0 & 0 & \cdots & 0 \\ \gamma_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \gamma_{L+n-1} & & \ddots & \gamma_0 \\ 0 & \ddots & & \gamma_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \gamma_{L+n-1} \end{bmatrix} \in \boldsymbol{R}^{(L+n+q)\times(q+1)}, \tag{37}$$

$$\begin{cases} e^* := f^* \bar{r} = \sum_{i=0}^{L+m+\ell-2} e^*[i] \lambda^i \\ \boldsymbol{e}^* := \begin{bmatrix} e^*[0] & e^*[1] & \cdots & e^*[L+m+\ell-2] & 0_{1\times(q+1)} \end{bmatrix}^T, \end{cases} \tag{38}$$

$$\begin{cases} u_k = \dfrac{D_P \bar{r} K_1^*}{1-\lambda^L} + D_P \bar{r} \bar{K}_1 = \dfrac{\beta}{1-\lambda^L} + \bar{\alpha} + D_P \bar{r} \bar{K}; \ \ \bar{\alpha}, \beta \in \boldsymbol{R}[\lambda] \\ \Delta u_k = \bar{\alpha} + D_P \bar{r} \bar{K} \quad \Delta u_k^* := \bar{\alpha} = \sum_{i=0}^{L+n-2} \Delta u_k^*[i] \lambda^i \\ \Delta \boldsymbol{u}_k^* := \begin{bmatrix} \Delta u_k^*[0] & \Delta u_k^*[1] & \cdots & \Delta u_k^*[L+n-2] & 0_{1\times(q+1)} \end{bmatrix}^T, \end{cases} \tag{39}$$

and

$$\phi := N\bar{r} = \sum_{i=0}^{L+m+\ell-1} \phi_i \lambda^i; \quad \gamma := D_P \bar{r} = \sum_{i=0}^{L+n-1} \gamma_i \lambda^i. \tag{40}$$

The minimum of J is given by

$$\min J = J_M - F_2^{\mathrm{T}} F_1^{-1} F_2, \tag{41}$$

where

$$J_M = \|\boldsymbol{e}^*\|_2^2 + \rho^2 \|\Delta \boldsymbol{u}_k^*\|_2^2 = \sum_{i=0}^{L+m+\ell-2} e^*[i]^2 + \rho^2 \sum_{i=0}^{L+n-2} \Delta u_k^*[i]^2. \tag{42}$$

5. NUMERICAL EXAMPLE

Consider the second-order plant

$$P(s) = \frac{\omega^2}{s^2 + 2\zeta(t)\omega s + \omega^2}. \tag{43}$$

We assume that

$$\begin{cases} \omega = 1(rad/s) \\ 0 \le \zeta(t) \le 1; \quad \zeta_0 = 0.5, \end{cases} \tag{44}$$

all the states are available, and there is no computational delay, i.e.

$$\kappa = 0. \tag{45}$$

Now we design a TDF repetitive controller which robustly stabilizes the control system and whose output tracks the periodic input

$$r(t) = \sin \frac{2\pi}{2.1} t + \sin \frac{4\pi}{2.1} t. \tag{46}$$

without steady-state error at the sampling points. The design is carried out under the following conditions:

$$\begin{cases} \tau = 0.1(s) \quad L = 21 \\ R^{1/2} = 0; \quad Q_p^{1/2} = C; \quad Q_k^{1/2} = I_{21\times 21} \\ q = 41 \quad \rho = 1. \end{cases} \tag{47}$$

The simulation results are shown in Figs. 6-9. In Fig. 6, ζ has the nominal value 0.5. As expected, the output reaches the steady state from the third cycle and tracks the reference input without steady state error. The control input during the transient is also moderately restricted. ζ =0 in Fig. 7 and ζ =1.0 in Fig. 8. It can be seen that, though the parameter of the plant is different from its nominal value, the system still remains stable and its output tracks the reference input without steady state error. And the control input is moderately restricted. For comparison, we plot the response of the minimal-settling-time controller $K_1 = K_1^*$ with $\zeta = 0.5$ in Fig. 9. It is clear that there were extreme changes in the control input during the transient.

6. CONCLUSIONS

This paper describes a design methodology for digital repetitive control systems in the case where there are some structured uncertainties in the plant. A TDF repetitive control system configuration is employed and a method of designing this system is presented. The advantage of the proposed method is that the feedback controller has the lowest order. The validity of the present method has been demonstrated by simulations.

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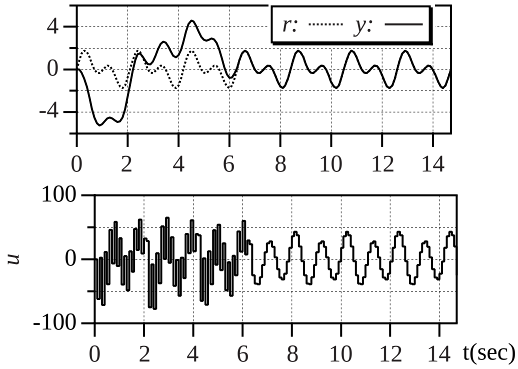

Fig. 6. Optimal response for $\zeta = 0.5$.

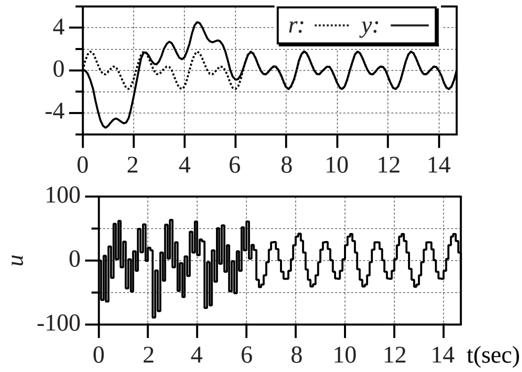

Fig. 7. Optimal response for $\zeta = 0$.

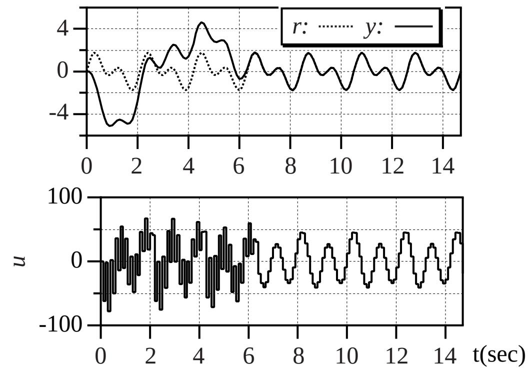

Fig. 8. Optimal response for $\zeta = 1.0$.

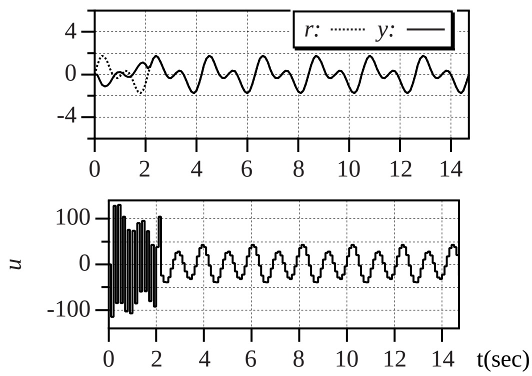

Fig. 9. Minimal settling - time response for $\zeta = 0.5$.