The 43rd IEEE Conference on Decision and Control, pp. 1736-1741, December 14-17, 2004, Atlantis, Paradise Island, Bahamas

Disturbance Estimation and Rejection — An equivalent input disturbance estimator approach —

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***Abstract*— This paper presents a new method of improving the disturbance rejection performance of a servo system by estimating an equivalent input disturbance. First, the concept of equivalent input disturbance is defined. Next, the configuration of an improved servo system employing the new disturbance estimation method is described. Then, a method of designing a control law employing the disturbance estimate is explained. Finally, the positioning control of a two-finger robot hand is used to demonstrate the validity of the method.**

I. INTRODUCTION

Over the past few decades, a considerable number of studies have been devoted to the estimation of an unknown disturbance [1]–[7]. While most of these methods require the differentiation of measured outputs, the methods in [1]–[4] do not. In [1] and [2], rank conditions are imposed on the unknown inputs; [3] requires information on the peak value of a disturbance; and [4] uses the inverse dynamics of the plant directly in the construction of the estimator. She and Ohyama proposed a new method that overcomes the drawbacks of these methods [8]. It requires neither the differentiation of measured outputs nor information on a disturbance, and does not use the inverse dynamics of the plant directly, thereby avoiding the cancellation of unstable poles/zeros. However, the state of the plant is needed for the estimation.

On the other hand, from the standpoint of the control system, it is more reasonable to estimate an equivalent input disturbance than to estimate the disturbance itself because we have to use the control input to improve the disturbance rejection performance.

This paper first defines an equivalent input disturbance for a system containing a disturbance that may not necessarily be imposed on the control input channel. Then, a new method of disturbance estimation based on the output of the plant is described. Finally, an improved servo system employing the disturbance estimate is constructed.

II. CONSTRUCTION OF IMPROVED SERVO SYSTEM

This section first defines an equivalent input disturbance, and then describes the configuration of an improved servo system constructed by inserting a disturbance estimator into a conventional servo system.

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A. Definition of equivalent input disturbance

Consider the linear time-invariant plant shown in Fig. 1.

$$\begin{cases} \dfrac{dx_o(t)}{dt} = Ax_o(t) + Bu(t) + B_d d(t), \\ y_o(t) = Cx_o(t), \end{cases} \quad (1)$$

where $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times n_u}$, $B_d \in \mathcal{R}^{n\times n_d}$, and $C \in \mathcal{R}^{n_y\times n}$. We consider the SISO case, which means that $n_u = 1$ and $n_y = 1$. Note that, since B and B_d may have different dimensions, the disturbance may be imposed on a channel other than that of the control input, and the number of disturbances and associated input channels may be larger than one. However, if we assume that a disturbance is imposed only on the control input channel, as shown in Fig. 2, then the plant is given by

$$\begin{cases} \dfrac{dx(t)}{dt} = Ax(t) + Bu(t) + Bd_e(t), \\ y(t) = Cx(t), \end{cases} \quad (2)$$

Then, an equivalent input disturbance is defined as follows:

Definition 1: Let the control input be $u(t) = 0$ and $x(\pm\infty) = 0$. Then, the output of the plant (1) for the disturbance $d(t)$ is $y_o(t)$, and the output of the plant (2) for the disturbance $d_e(t)$ is $y(t)$. The disturbance $d_e(t)$ is called an equivalent input disturbance of the disturbance $d(t)$ if $y(t) = y_o(t)$ for all $t \geq 0$.

The following assumption is made about the plant.

Assumption 1: (A, B) is controllable and (C, A) is observable.

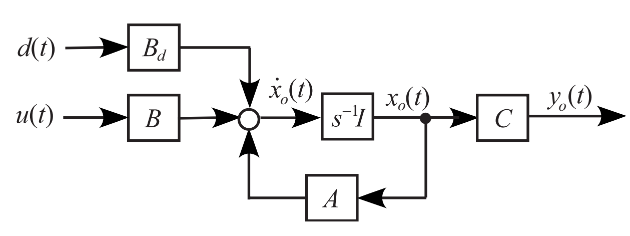

Fig. 1. Plant.

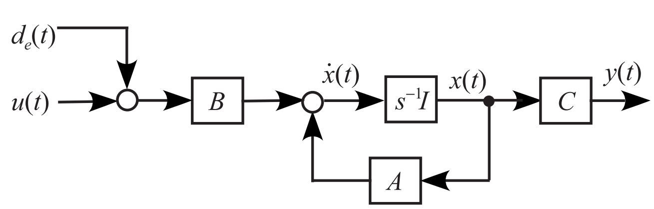

Fig. 2. Plant with an equivalent input disturbance.

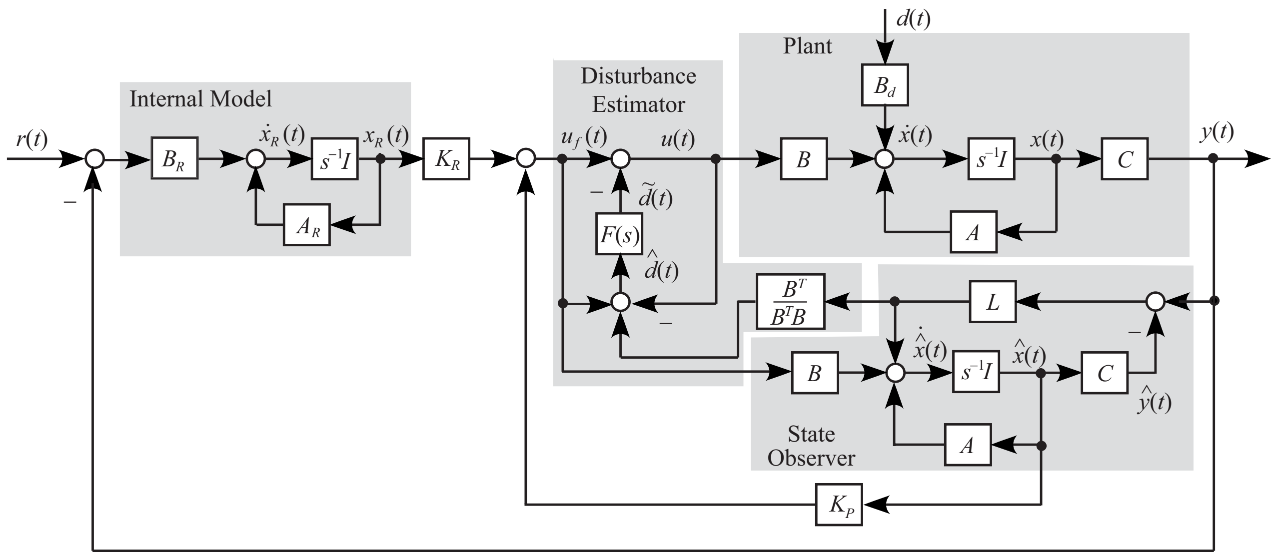

Fig. 3. Configuration of improved servo system.

If the trajectory of the output caused by the disturbance $d(t)$ is $y_d(t) \in L_1 \cap L_\infty$, then from the concept of stable inversion [9], [10], it is known that there exists an equivalent disturbance, $d_e(t) \in L_1 \cap L_\infty$, on the control input channel that produces the same trajectory. This leads to the following lemma.

Lemma 1: There always exists an equivalent input disturbance, $d_e(t) \in L_1 \cap L_\infty$, on the control input channel of the disturbance, $d(t)$, that is imposed on the plant (1); and the output it produces belongs to $L_1 \cap L_\infty$.

B. Estimation of equivalent input disturbance

The configuration of an improved servo system is shown in Fig. 3. It can be viewed as a conventional servo system (internal model, state observer and state feedback) combined with a disturbance estimator that produces an estimate of an equivalent input disturbance. K_R and K_P are the state-feedback gains; L is the observer gain; and $F(s)$ is a low-pass filter that limits the angular frequency band of the disturbance estimate.

In Fig. 3, for the state observer,

$$\frac{d\hat{x}(t)}{dt} = A\hat{x}(t) + Bu_f(t) + LC[x(t) - \hat{x}(t)] \tag{3}$$

holds. Letting the estimate of the equivalent input disturbance be $\hat{d}(t)$ allows us to write

$$\frac{d\hat{x}(t)}{dt} = A\hat{x}(t) + B[u(t) + \hat{d}(t)] \tag{4}$$

for the plant with an equivalent input disturbance. Without loss of generality, we assume $B^T B \neq 0$. Then, (3) and (4) yield

$$\hat{d}(t) = \frac{B^T}{B^T B} LC[x(t) - \hat{x}(t)] + u_f(t) - u(t). \tag{5}$$

$\hat{d}(t)$ is filtered by $F(s)$, which selects the angular frequency band for the disturbance estimation. Thus, the filtered disturbance estimate, $\tilde{d}(t)$, is given by

$$\tilde{D}(s) = F(s)\hat{D}(s), \tag{6}$$

where $\tilde{D}(s)$ and $\hat{D}(s)$ are the Laplace transforms of $\tilde{d}(t)$ and $\hat{d}(t)$, respectively.

Remark 1: Since an estimate is obtained for an equivalent input disturbance, the channel on which the equivalent disturbance is imposed might be different from that of the actual disturbance. So, generally speaking, a full state observer must be used to estimate the state of the plant. And if a disturbance exists, then the estimated state of the plant might be different from the actual state resulting from the effects of the disturbance. Looking at this from a different perspective, in (4), we can take the state of the plant with an equivalent input disturbance to be $\hat{x}(t)$, which is exactly the state of the observer, and consider the difference between the output of the real plant and that of the plant with an equivalent input disturbance to be caused by the difference between the exact value and the estimate of the equivalent input disturbance.

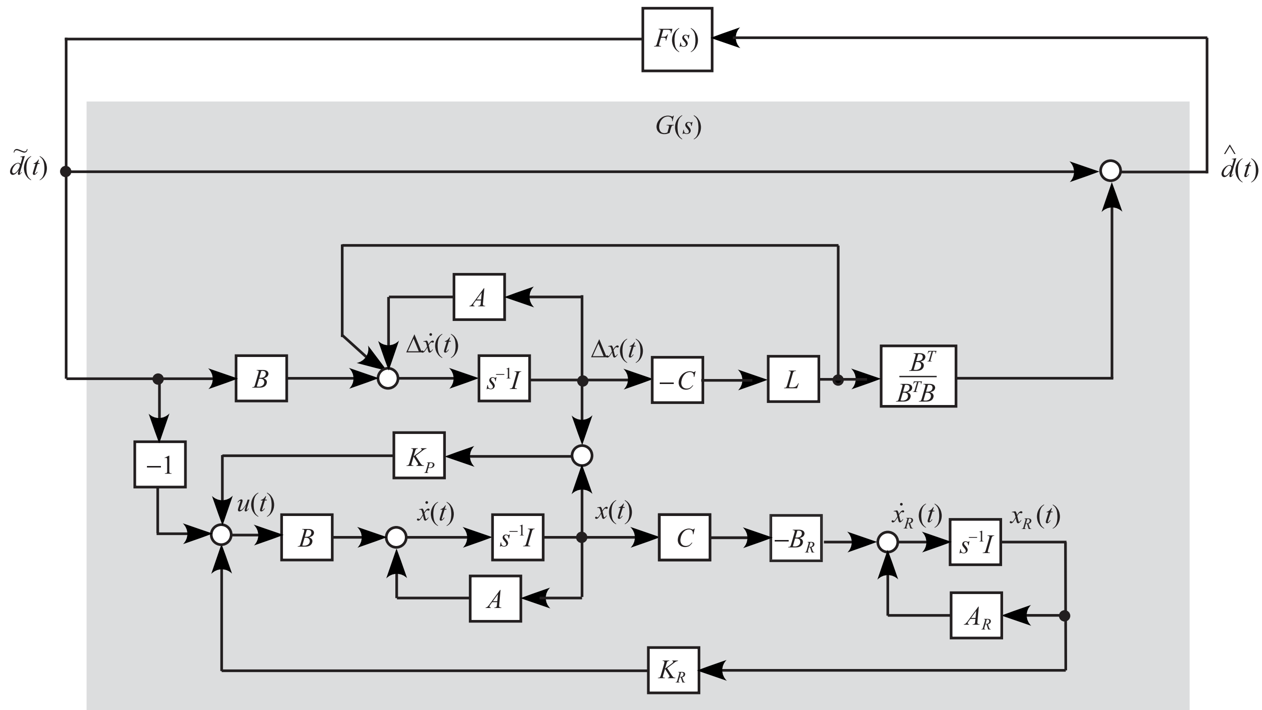

Fig. 4. Block diagram for design of low-pass filter and state observer.

C. Disturbance rejection

Combining the disturbance estimate (6) with the original servo control law yields the following control law:

$$u(t) = u_f(t) - \tilde{d}(t), \tag{7}$$

as shown in Fig. 3. This modified control law improves the disturbance rejection performance. The method described in this paper has two important characteristics not provided by previous methods:

1) The configuration of the system is very simple.
2) The disturbance rejection performance can easily be improved by incorporating the disturbance estimate directly into the designed servo control law.

Regarding the first characteristic, the improved servo system can be viewed as a conventional servo system enhanced by the plugging-in of a disturbance estimate. So, the structure is very simple and very easy to understand. Regarding the second characteristic, a suitable design of the observer guarantees that $\hat{d}(t)$ converges to $d_e(t)$, and $|\hat{d}(t) - \tilde{d}(t)| < |\hat{d}(t)|$ is guaranteed by a properly designed low-pass filter, $F(s)$. So, $\tilde{d}(t)$ is a good approximation of $d_e(t)$.

This system also has another very important characteristic: the state-feedback control law, and the observer and low-pass filter can be designed independently, as long as stability is the only concern. This is discussed in the following subsection.

D. Design of filter and state observer

The state observer gain, L, and the low-pass filter, $F(s)$, should be designed so that they do not destroy the stability of the system. Regarding the stability issue, the system with $r(t) = 0$, $d(t) = 0$ and $\Delta x(t) = \hat{x}(t) - x(t)$ is illustrated in Fig. 4. The plant is

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t). \tag{8}$$

Combining (3) and (7) with the above equation yields

$$\frac{d\Delta x(t)}{dt} = (A - LC)\Delta x(t) + B\tilde{d}(t). \tag{9}$$

On the other hand, (5) is equivalent to

$$\hat{d}(t) = -\frac{B^T}{B^T B} LC\Delta x(t) + \tilde{d}(t). \tag{10}$$

(9) and (10) yield the transfer function from $\tilde{d}(t)$ to $\hat{d}(t)$:

$$\begin{aligned} G(s) &= 1 - \frac{B^T}{B^T B} LC[sI - (A - LC)]^{-1} B \\ &= \frac{B^T (sI - A)[sI - (A - LC)]^{-1} B}{B^T B}. \end{aligned} \tag{11}$$

Thus, we obtain the following from the small-gain theorem.

Theorem 1: For a suitably designed state-feedback gain, $[K_P \ K_R]$, the control law (7) guarantees the stability of the control system if

$$\|G(j\omega)F(j\omega)\|_\infty < 1, \ \forall\omega \in [0, \ \infty). \tag{12}$$

Remark 2: The stability conditions for the improved servo system can be broken down into two parts. First, the state-feedback servo system is stable. Second, condition (12) holds. Since the only parameters in condition (12) are L and $F(s)$, their design is much simpler than that in the disturbance observer method [4], which requires a low-pass filter to guarantee the stability of the whole system.

In the angular frequency band for disturbance rejection,

$$\Omega_r = \{\omega : \ \omega \leq \omega_r\},$$

it is most desirable to choose a low-pass filter, $F(s)$, to be

$$|F(j\omega)| \leq 1, \ \forall \omega \in \Omega_r. \tag{13}$$

So, we have to choose an observer gain, L, such that

$$\|G(j\omega)\|_\infty < 1, \ \forall \omega \in \Omega_r. \tag{14}$$

On the other hand, for the system

$$\begin{cases} \dfrac{dx_L}{dt} = A^T x_L + C^T u_L, \\ y_L = B^T x_L, \end{cases} \tag{15}$$

consider state feedback parameterized by a scalar, $\rho \geq 0$:

$$u_L = L_\rho^T x_L.$$

If (A^T, C^T, B^T) is a minimum-phase system, then we can obtain an L_ρ^T that provides perfect regulation [11], [12], and

$$\lim_{\rho \to \infty} [sI - (A - L_\rho C)]^{-1} B = 0$$

holds. Note that $[sI - (A - L_\rho C)]^{-1}B$ is part of the numerator of $G(s)$, which means that a large enough ρ makes $|G(j\omega)|$ sufficiently small for all $\omega \in \Omega_r$. So, based on the concept of perfect regulation, we can obtain an L and $F(s)$ that satisfy the condition (12). The design procedure is explained in the next subsection.

E. Design procedure

Summarizing the above results, we can now give a design algorithm for the improved servo system.

Design algorithm:

Step 1. Design the feedback gains K_P and K_R for a conventional servo system using an existing method (for example, the optimal control method).

Step 2. Choose an angular frequency band, Ω_r, for disturbance rejection.

Step 3. Select a large enough ρ that yields an L that makes (14) true.

Step 4. Plot $1/G(j\omega)$, and select a $F(s)$ that satisfies $|F(j\omega)| < |1/G(j\omega)|$ for all $\omega \in [0, \infty)$ based on the gain characteristics of its Bode plot.

III. Numerical Example

We employed the method described above for the positioning control of a two-finger robot hand [13] with the structure shown in Fig. 5. One of the fingers is immobile, and the other is driven by a DC motor and a belt-pulley mechanism. The dynamics of the hand can be derived using Lagrange's equation.

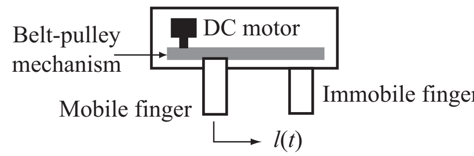

Fig. 5. Structure of two-finger robot hand.

$$M\frac{d^2 l(t)}{dt^2} + K_f \frac{dl(t)}{dt} = f(t) + d_f(t),$$

where $l(t)$ [m] is the distance that the finger moves; M [kg] is the mass of the moving part; K_f [kg/s] is the coefficient of viscous friction; and $f(t)$ [N] and $d_f(t)$ [N] are the driving forces produced by the motor and the disturbance force, respectively. When the dynamics of the electric circuit of the motor cannot be ignored, the dynamics have the form

$$\begin{aligned} &R_e i(t) + L_e \frac{di(t)}{dt} = u_M(t) - K_E \frac{dl(t)}{dt} + d_u(t), \\ &f(t) = K_T i(t), \end{aligned}$$

where R_e [Ω] is the resistance of the motor, L_e [H] is the inductance of the motor, K_E [Vs/m] is the back electromotive force constant, K_T [N/A] is the force constant, $i(t)$ [A] is the armature current, $u(t)$ [V] is the control voltage, and $d_u(t)$ [V] is the disturbance voltage. Choosing the state to be $x_o(t) = [l(t)\ dl(t)/dt\ i(t)]^T$ yields the following values for the parameters in (1).

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -K_f/M & K_T/M \\ 0 & -K_E/L_e & -R_e/L_e \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0 \\ 1/L_e \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0 & 0 \\ 1/M & 0 \\ 0 & 1/L_e \end{bmatrix}, \ C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

The fact that this plant is a minimum-phase system allows us to use the proposed method to design an observer gain and a low-pass filter. We assume that $M = 1$, $K_f = 1$, $R_e = 2$, $L_e = 1$, $K_T = 1$, and $K_E = 1$. Then, we let the reference input be the unit step signal

$$\begin{cases} r(t) = 1(t), \\ A_R = 0, \ B_R = 1, \end{cases} \tag{16}$$

and the disturbances be

$$d_u(t) = \begin{cases} 0, & t < 10, \\ 5\sin 2\pi t + 2.5 \sin 4\pi t + 1.25 \sin 6\pi t, & t \geq 10; \end{cases}$$

$$d_f(t) = \begin{cases} 0, & t < 10, \\ 0.2 \sin 2\pi t - 2\tanh t + 4\tanh(t - 10), & t \geq 10. \end{cases} \tag{17}$$

The improved servo system was designed by following the design procedure in the previous section. First, the disturbances were ignored; and a single augmented-state

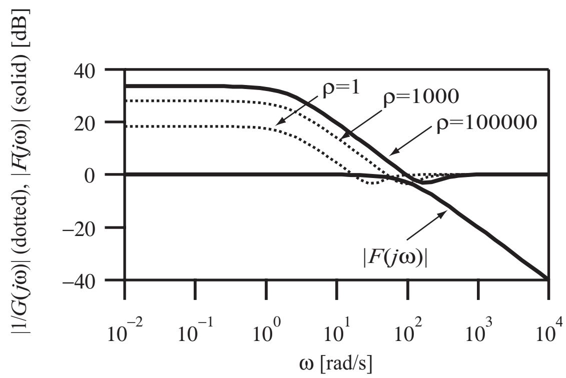

Fig. 6. Tuning of $G(s)$ and selection of $F(s)$.

representation containing the plant and an internal model of the step signal [14] was constructed:

$$\frac{d}{dt}\begin{bmatrix}\delta x(t)\\ \delta x_R(t)\end{bmatrix}=\begin{bmatrix}A_P & 0\\ -C_P & 0\end{bmatrix}\begin{bmatrix}\delta x(t)\\ \delta u_f(t)\end{bmatrix}+\begin{bmatrix}B_P\\ 0\end{bmatrix}\delta u_f(t), \tag{18}$$

where

$$\delta x(t)=x(t)-x(+\infty),$$
$$\delta x_R(t)=x_R(t)-x_R(+\infty),$$
$$\delta u_f(t)=u_f(t)-u_f(+\infty).$$

Minimizing the performance index

$$J_K=\int_0^{\infty}\left\{\left[\delta x^T(t)\ \delta x_R^T(t)\right]Q_K\begin{bmatrix}\delta x(t)\\ \delta x_R(t)\end{bmatrix}+R_K\delta u_f^2(t)\right\}dt,$$
$$Q_K=\mathrm{diag}\{100\ 1\ 1\ 100\},$$
$$R_K=1$$

yields

$$[K_P\ K_R]=[-19.9779\ -9.13249\ -2.82338\ 10.0000]\,.$$

Next, Ω_r was selected by setting ω_r to $100\ \mathrm{rad/s}$. Then, an optimal filter gain, L, was designed that minimized the performance index

$$J_L=\int_0^{\infty}\left\{\rho x_L^T(t)Q_L x_L(t)+R_L u_L^2(t)\right\}dt,$$
$$Q_L=\mathrm{diag}\{1\ 1\ 10^9\},$$
$$R_L=1$$

for the system (15). ρ was adjusted so that

$$\|G(j\omega)\|_\infty<1,\ \forall\omega\in\Omega_r.$$

The result was $\rho=10^5$, which yielded

$$L=\left[\ 554.919\quad 103967\quad 9790401\ \right]^T.$$

Finally, the low-pass filter

$$F(s)=\frac{1}{Ts+1},\ T=0.01$$

was chosen. It is clear from Fig. 6 that the stability condition (12) is satisfied. So, the improved servo system is stable.

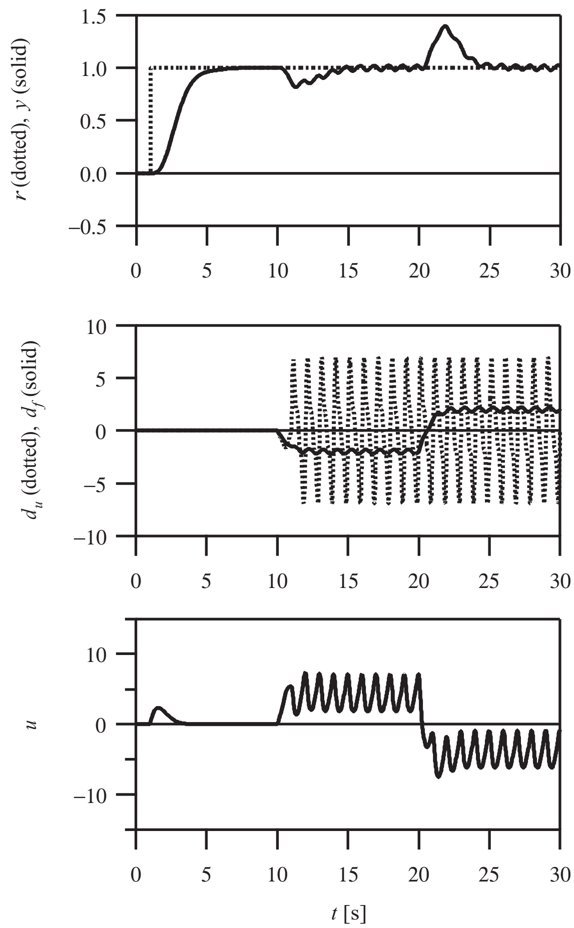

Fig. 7. Simulation results without disturbance estimation.

Some simulation results are shown in Figs. 7 and 8. In Fig. 7, the step reference input is imposed at $t=1\ \mathrm{s}$. After the system enters the steady state, the disturbances (17) are imposed on the system starting at $t=10\ \mathrm{s}$.

Since servo control just suppresses the disturbances to some extent but cannot reject them, a large steady-state tracking error (peak-to-peak value: 0.04401) results. The corresponding control input is also shown in the same figure. On the other hand, a big improvement is obtained when the proposed method is employed. The simulation results obtained with disturbance estimation are shown in Fig. 8. It can be seen that the disturbance is rejected satisfactorily in both the transient and steady states. The steady-state tracking error drops to 0.00291 (peak-to-peak), which is less than 7% of that without disturbance estimation. The estimated equivalent input disturbance is also shown in the same figure. It increases the control input, thereby suppressing the effects of the disturbances.

IV. Conclusions

This paper describes a new method of improving the disturbance rejection performance of a servo system by estimating an equivalent input disturbance. Based on this

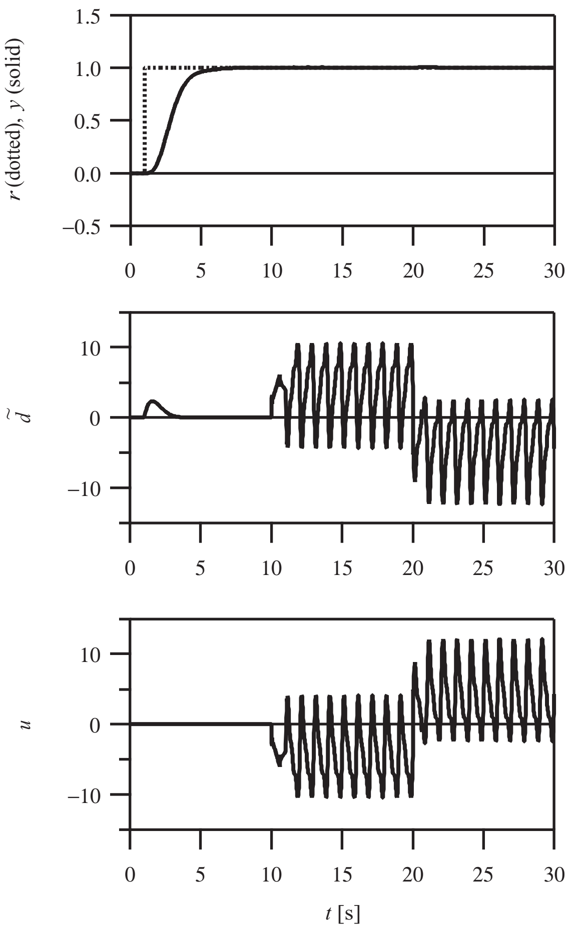

Fig. 8. Simulation results with disturbance estimation.

method, an improved servo system configuration was devised. The proposed method has some significant advantages over existing methods:

- It does not require the differentiation of measured outputs.
- It avoids cancellation of unstable poles/zeros.
- The stability of the system can be broken down into two independent parts: state feedback, and observer and low-pass filter.
- The configuration is very simple.

The validity of the method was demonstrated using the positioning control of a two-finger robot as an example.

REFERENCES

[1] M. Corless and J. Tu, "State and Input Estimation for a Class of Uncertain Systems", *Automatica*, vol. 34, no. 6, pp. 757-764, 1998.
[2] M. Hou and R.J. Patton, "Optimal Filtering for Systems with Unknown Inputs", *IEEE Trans. Autom. Contr.*, vol. 43, no. 3, pp. 445-449, 1998.
[3] İ. Haskara and Ü. Özuüner, "An Estimation Based Robust Tracking Controller Design for Uncertain Nonlinear Systems", *Proc. 38th IEEE Conf. Decision & Control*, pp. 4816-4821, 1999.
[4] M. Tomizuka, "Model Based Prediction, Preview and Robust Controls in Motion Control Systems", *Proc. 4th International Workshop on Advanced Motion Control*, vol. 1, pp. 1-6, 1996.
[5] C.-S. Liu and H. Peng, "Inverse-Dynamics Based State and Disturbance Observers for Linear Time-Invariant Systems", *Trans. ASME J. Dynamic Systems, Measurement, and Control*, vol. 124, no. 3, pp. 375-381, 2002.
[6] Y. Xiong and M. Saif, "Sliding Mode Observer for Nonlinear Uncertain Systems", *IEEE Trans. Autom. Contr.*, vol. 46, no. 12, pp. 2012-2017, 2001.
[7] X. Chen, T. Fukuda and K.D. Young, "A new nonlinear robust disturbance observer", *Syst. Contr. Lett.*, vol. 41, no. 3, pp. 189-199, 2000.
[8] J.-H. She and Y. Ohyama, "Estimation and Rejection of Disturbances in Servo Systems", *Proc. 15th IFAC World Congress*, vol. S, pp. 67-72, 2002.
[9] L.R. Hunt, G. Meyer and R. Su, "Noncausal Inverses for Linear Systems", *IEEE Trans. Autom. Contr.*, vol. 41, no. 4, pp. 608-611, 1996.
[10] S. Devasia, D. Chen and B. Paden, "Nonlinear Inversion-Based Output Tracking", *IEEE Trans. Autom. Contr.*, vol. 41, no. 7, pp. 930-942, 1996.
[11] H. Kwakernaak and R. Sivan, "The Maximally Achievable Accuracy of Linear Optimal Regulators and Linear Optimal Filters", *IEEE Trans. Autom. Contr.*, vol. 17, no. 1, pp. 79-86, 1972.
[12] H. Kimura, "A New Approach to the Perfect Regulation and the Bounded Peaking in Linear Multivariable Control Systems", *IEEE Trans. Autom. Contr.*, vol. 26, no. 1, pp. 253-270, 1981.
[13] R. Suzuki, S. Hasegawa and N. Kobayashi, "Combining IMC Design with H_∞ Control and Its Application to A Two-Finger Robot Hand", *Proc. 2001 IEEE Int. Conf. on Control Applications*, pp. 1077-1082, 2001.
[14] F.L. Lewis, *Applied Optimal Control & Estimation –Digital Design & Implementation–*, Englewood Cliffs, NJ: Prentice Hall and Texas Instruments, 1992.