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Motion Control of Acrobots Using Fuzzy and Sliding-Mode Control Strategy

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Abstract

This paper describes a control strategy for the control of an acrobot. The strategy combines a model-free fuzzy control, a fuzzy sliding-mode control and a model-based fuzzy control. The model-free fuzzy controller designed for the upswing ensures that the energy of the acrobot increases with each swing. Then the fuzzy sliding-mode controller is employed to control the movement that the acrobot enters the attractive area from the swing-up area. The model-based fuzzy controller, which is based on a Takagi-Sugeno fuzzy model is used to balance the acrobot. The stability of the fuzzy control system for balance control is guaranteed by a common symmetric positive matrix, which satisfies linear matrix inequalities.

1. Introduction

The acrobot is a two-link manipulator operating in a vertical plane with an actuator at the elbow but no actuator at the shoulder. It is a good example of an underactuated mechanical system, which possesses fewer actuators than the degrees of freedom [1].

Motion control of an acrobot has been studied for years. For example, [2] has investigated the problem of balancing an acrobot at the unstable straight-up position using nonlinear approximation. In [3, 4], Spong has described a partial feedback linearization method to swing an acrobot up and has used the linear quadratic regulator (LQR) method to balance it. However, the upswing control law was chosen based on the condition under which the energy of a single link increases. So theoretically, it did not guarantee the energy of the acrobot increased with each swing. In addition, the capture of the acrobot is very difficult and the LQR balancing control law[3, 4] makes the region for balance control very small.

This paper proposes a control strategy that employs a model-free fuzzy control, a fuzzy sliding-mode control and a model-based fuzzy control. In the swing-up process, the control law for the torque is derived directly from the energy of the acrobot, and the model-free fuzzy controller regulates the amplitude of the control torque according to the energy. The key point is to choose a control torque that guarantees that the energy of the acrobot increases with each swing. This is quite different from the method proposed in [4]. Then the fuzzy sliding-mode controller is employed to control the second link of the acrobot. This control law drives the angle of the second link towards zero, and maintains the energy of the acrobot almost unchanged. This strategy ensures that the acrobot enters the attractive area quickly, and overcomes the difficulty of the capture of the acrobot in the swing-up area. It guarantees that the acrobot enters the attractive area very fast and easily. In the balancing process, a Takagi-Sugeno fuzzy model is constructed to approximate the dynamics of the acrobot. The model-based fuzzy controller, which uses the Takagi-Sugeno fuzzy model, employs the concept of parallel distributed compensation. The stability of the fuzzy control system for balance control is guaranteed by a common symmetric positive matrix, which satisfies linear matrix inequalities (LMIs). Since the Takagi-Sugeno fuzzy model describes the acrobot with a satisfactory approximated precision over a large region, the model-based fuzzy balancing control law makes the attractive area larger than it is with LQR.

2. **Dynamics of the acrobot**

Consider the acrobot shown in Figure 1. Its dynamic equations are

$$m_{11}(\mathbf{q})\ddot{q}_1 + m_{12}(\mathbf{q})\ddot{q}_2 + c_1(\mathbf{q},\ \dot{\mathbf{q}}) + g_1(\mathbf{q}) = 0\,, \tag{1a}$$

$$m_{21}(\mathbf{q})\ddot{q}_1 + m_{22}(\mathbf{q})\ddot{q}_2 + c_2(\mathbf{q},\ \dot{\mathbf{q}}) + g_2(\mathbf{q}) = \tau\,, \tag{1b}$$

where

$\mathbf{q} = [q_1 \quad q_2]^T$,

$m_{11}(\mathbf{q}) = m_1 L_{g1}^2 + I_1 + m_2 L_{g2}^2 + I_2 + m_2 L_1^2 + 2 m_2 L_1 L_{g2} \cos q_2$,

$m_{22}(\mathbf{q}) = m_2 L_{g2}^2 + I_2$,

$m_{12}(\mathbf{q}) = m_{21}(\mathbf{q}) = m_2 L_{g2}^2 + I_2 + m_2 L_1 L_{g2} \cos q_2$,

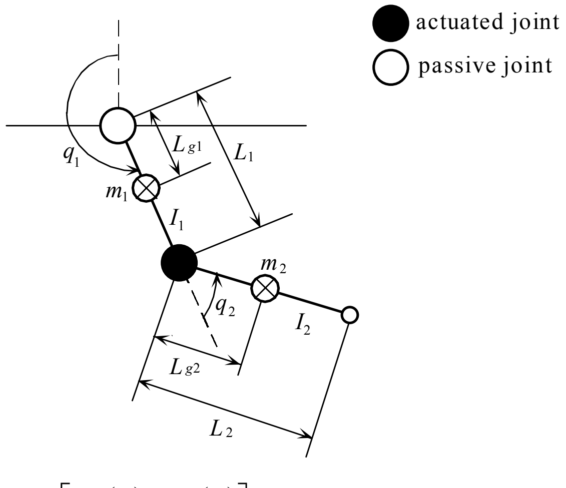
$c_1(\mathbf{q},\ \dot{\mathbf{q}}) = -m_2 L_1 L_{g2} \dot{q}_2 (2\dot{q}_1 + \dot{q}_2) \sin q_2$,

$c_2(\mathbf{q},\ \dot{\mathbf{q}}) = m_2 L_1 L_{g2} \dot{q}_1^2 \sin q_2$,

$g_1(\mathbf{q}) = -(m_1 L_{g1} + m_2 L_1) g \sin q_1 - m_2 L_{g2} g \sin(q_1 + q_2)$,

$g_2(\mathbf{q}) = -m_2 L_{g2} g \sin(q_1 + q_2)$.

For the link i (i=1,2), the parameters q_i, $\dot{q}_i$, m_i, L_i, L_{gi} and I_i are the angle, the angular velocity, the mass, the link length, the center of mass, and the moment of inertia, respectively. The inertia matrix $\mathbf{M}(\mathbf{q})$ is



$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_{11}(\mathbf{q}) & m_{12}(\mathbf{q}) \\ m_{21}(\mathbf{q}) & m_{22}(\mathbf{q}) \end{bmatrix}\,, \tag{2}$$

Figure 1. Model of the acrobot.

In this paper, the motion space of the acrobot is divided into two subspaces[5]: one is the attractive area in the neighborhood of the unstable straight-up equilibrium position, and the remainder is the swing-up area. Two small positive numbers, λ_1 and λ_2, are used to define the two subspaces.

Swing-up area: $|q_1| > \lambda_1$ and $|q_1 + q_2| > \lambda_2$, (3)

Attractive area: $|q_1| \le \lambda_1$ or $|q_1 + q_2| \le \lambda_2$. (4)

3. Control in the swing-up area

In the swing-up area, the motion control of the acrobot includes two phases. In the first phase, the control torque is derived directly from the energy of the acrobot. A model-free fuzzy controller is designed to regulate its amplitude of the control torque according to the energy. It is employed until the energy reaches the amount that the acrobot has at the unstable straight-up equilibrium position. In the second phase, a fuzzy sliding-mode controller is employed to control the second link. It drives the angle of the second link towards zero, and maintains the energy almost unchanged. This strategy ensures that the acrobot enters the attractive area quickly, and overcomes the difficulty of the capture of the acrobot in the swing-up area.

3.1.Design of model-free fuzzy controller

The energy of the acrobot is given by

$$E(\mathbf{q},\ \dot{\mathbf{q}}) = T(\mathbf{q},\ \dot{\mathbf{q}}) + V(\mathbf{q}), \tag{5}$$

where $T(\mathbf{q},\ \dot{\mathbf{q}})$ is the kinetic energy and $V(\mathbf{q})$ is the potential energy, both of which are expressed in generalized coordinates. They are defined as follows:

$$T(\mathbf{q},\ \dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q})\dot{\mathbf{q}}, \tag{6}$$

$$V(\mathbf{q}) = \sum_{i=1}^{2} V_i(\mathbf{q}) = \sum_{i=1}^{2} m_i g h_i(\mathbf{q}), \tag{7}$$

where $V_i(q)$ and $h_i(q)$ are the potential energy and the height of the center of mass of the ith link, respectively.

During an upswing, the energy of the acrobot should increase continuously until it reaches the amount that the acrobot has at the unstable straight-up equilibrium position. This means that the derivation of the energy should satisfy the following condition in the swing-up area.

$$\dot{E}(\mathbf{q},\ \dot{\mathbf{q}}) \ge 0\,. \tag{8}$$

Differentiating (5) and (6) to $\mathbf{q}$ and $\dot{\mathbf{q}}$, (7) to $\mathbf{q}$ respectively, and rewriting the dynamic equation (1) yield

$$\dot{E}(\mathbf{q},\ \dot{\mathbf{q}}) = \dot{q}_2 \tau\,. \tag{9}$$

So, the control torque for swing-up is chosen to be

$$\tau = \mathrm{sgn}(\dot{q}_2)\upsilon\,, \qquad \upsilon \ge 0 \tag{10}$$

to satisfy (8).

The control variable υ in (10) can be chosen arbitrarily in the admissible range of the control torque as long as it is positive. Clearly, the amplitude of the control torque should be chosen so that it decreases as the energy increases. That makes the acrobot can work smoothly when the control law changes. To implement this strategy, a model-free fuzzy controller is designed to determine the control variable υ.

A basic fuzzy control method[6] is used to design the model-free fuzzy controller. The input of the model-free fuzzy controller is the energy $E(\mathbf{q}, \dot{\mathbf{q}})$ and the fuzzy output variable is υ_x. The fuzzy relation between the energy $E(\mathbf{q}, \dot{\mathbf{q}})$ and the fuzzy output variable υ_x is the set of simple fuzzy rules as following

If $E(\mathbf{q}, \dot{\mathbf{q}})$ is small, Then υ_x is large;

If $E(\mathbf{q}, \dot{\mathbf{q}})$ is medium, Then υ_x is medium;

If $E(\mathbf{q}, \dot{\mathbf{q}})$ is large, Then υ_x is small.

The membership functions (mfs) for fuzzy input/output linguistic variables are chosen to have the triangular shapes. The crisp output, υ is obtained by applying the center-of-gravity defuzzification method to the fuzzy output variable υ_x. The model-free fuzzy controller is employed until the energy reaches the amount that the acrobot has at the unstable straight-up equilibrium position. Then the fuzzy sliding-mode controller is applied to control the angle of the second link towards zero, and maintains the energy unchanged.

3.2.Design of fuzzy sliding-mode controller

Dynamic equations (1) and (2) can be rewritten as

$$\dot{x}_1 = x_3, \tag{11a}$$

$$\dot{x}_2 = x_4, \tag{11b}$$

$$\dot{x}_3 = f_1(\mathbf{x}) + b_1(\mathbf{x})\tau, \tag{11c}$$

$$\dot{x}_4 = f_2(\mathbf{x}) + b_2(\mathbf{x})\tau, \tag{11d}$$

where $\mathbf{x} = [x_1 \ \ x_2 \ \ x_3 \ \ x_4]^T = [q_1 \ \ q_2 \ \ \dot{q}_1 \ \ \dot{q}_2]^T$ is the state vector, $f_1(\mathbf{x})$, $b_1(\mathbf{x})$, $f_2(\mathbf{x})$ and $b_2(\mathbf{x})$ are nonlinear functions.

The sliding function is defined as

$$s = cx_2 + x_4, \qquad c > 0, \tag{12}$$

the sliding surface is

$$cx_2 + x_4 = 0. \tag{13}$$

Choosing a lyapunov function

$$V = \frac{1}{2}s^2. \tag{14}$$

From the lyapunov stability theorem, if $\dot{V}$ is negative, the states of the second link, x_2 and x_4, will be driven and attracted towards the sliding surface and remain sliding on it until the states of the second link asymptotically converge to zeroes. From (12) and (14), the following expression is obtained

$$\dot{V} = s[cx_4 + f_2(\mathbf{x}) + b_2(\mathbf{x})\tau]. \tag{15}$$

Satisfy $\dot{V}$ being negative, the control torque is chosen as

$$\tau = \tilde{\tau} - K\,\mathrm{sgn}(sb_2(\mathbf{x})), \quad K \geq 0, \tag{16}$$

where K must be properly chosen, and

$$\tilde{\tau} = \frac{-cx_4 - f_2(\mathbf{x})}{b_2(\mathbf{x})}. \tag{17}$$

The controller will have high frequency switching chattering near the sliding surface due to signum function involved. These drastic changes of input can be avoided by introducing a boundary layer with width Φ. Thus, replacing $\mathrm{sgn}(sb(\mathbf{x}))$ with $\mathrm{sat}(sb(\mathbf{x})/\Phi)$, the law of the sliding mode control is

$$\tau = \tilde{\tau} - K\,\mathrm{sat}(sb_2(\mathbf{x})/\Phi), \tag{18}$$

where

$$\mathrm{sat}(sb_2(\mathbf{x})/\Phi) = \begin{cases} \mathrm{sgn}(sb_2(\mathbf{x})/\Phi) & \text{if } |sb_2(\mathbf{x})/\Phi| \geq 1 \\ sb_2(\mathbf{x})/\Phi & \text{if } |sb_2(\mathbf{x})/\Phi| < 1 \end{cases}. \tag{19}$$

Substituting (18) into (9) yields

$$\dot{E}(q, \dot{q}) = \dot{q}_2\tilde{\tau} - K\dot{q}_2\,\mathrm{sat}(sb_2(\mathbf{x})/\Phi), \tag{20}$$

$\dot{E}(\mathbf{q}, \dot{\mathbf{q}})$ is not always equal to zero, which means that the energy changes when the sliding mode control is employed.

Maintaining the energy unchanged, K is chosen as

$$K = \beta(1+\rho), \quad \beta > 0, \quad -1 < \rho < 1. \tag{21}$$

Two fuzzy controllers are designed according to $y = -\dot{q}_2\,\mathrm{sat}(sb_2(\mathbf{x})/\Phi) \geq 0$ and $y < 0$ to regulate the parameter K. Two inputs of the fuzzy controller are the energy change

$$e = T(\mathbf{q}, \dot{\mathbf{q}}) + V(\mathbf{q}) - E \tag{22}$$

and the signal w (where $w = \dot{q}_2\tilde{\tau}$), respectively, and the output is ρ', where E is the amount of the energy that the acrobot has at the unstable equilibrium position.

The fuzzy relations between the inputs and output are listed in Table 1 and 2, where NB is negative big, NM is negative medium, ZR is zero, PM is positive medium, and PB is positive big. The fuzzy rules are equivalent to

Rule l: IF e is $\tilde{F}_e^l$ and w is $\tilde{F}_w^l$, THEN ρ' is $\tilde{F}_{\rho'}^l$,

$$l = 1, 2, \ldots, r, \qquad (23)$$

where r is the number of IF-THEN rules. $\tilde{F}_e^l$, $\tilde{F}_w^l$ and $\tilde{F}_\rho^l$ are the fuzzy sets of e, w and ρ', respectively.

Table 1 Fuzzy control rules $y \geq 0$

w \ e	NB	NM	ZR	PM	PB
NB	PB	PB	PB	PM	ZR
NM	PB	PB	PM	ZR	NM
ZR	PB	PM	ZR	NM	NB
PM	PM	ZR	NM	NB	NB
PB	ZR	NM	NB	NB	NB

Table 2 Fuzzy control rules $y < 0$

w \ e	NB	NM	ZR	PM	PB
NB	NB	NB	NB	NM	ZR
NM	NB	NB	NM	ZR	PM
ZR	NB	NM	ZR	PM	PB
PM	NM	ZR	PM	PB	PB
PB	ZR	PM	PB	PB	PB

Both of the fuzzy controllers use the (Product, Product, Maxim, Center of Gravity) fuzzy operation.

The compositional degree ω_l of the fuzzy control premise condition is

$$\omega_l = \mu_{\tilde{F}_e^l}(e) * \mu_{\tilde{F}_w^l}(w). \qquad (24)$$

The inference result of the lth fuzzy rule is

$$\mu_{\tilde{F}_{\rho'}}(\zeta) = \omega_l * \mu_{\tilde{F}_{\rho'}^l}(\zeta), \qquad (25)$$

where $\mu_{\tilde{F}_e^l}(e)$, $\mu_{\tilde{F}_w^l}(w)$ and $\mu_{\tilde{F}_{\rho'}^l}(\zeta)$ are the membership functions of the fuzzy sets $\tilde{F}_e^l$, $\tilde{F}_w^l$ and $\tilde{F}_{\rho'}^l$, respectively, $*$ is the Product.

The compositional inference consequence of all rules is

$$\mu_{\tilde{F}_\rho}(\zeta) = \bigvee_{l=1}^{25} \omega_l * \mu_{\tilde{F}_{\rho'}^l}(\zeta). \qquad (26)$$

The crisp output ρ is obtained by the center-of-gravity defuzzifier

$$\rho = \frac{\int \mu_{\tilde{F}_{\rho'}}(\rho')\rho' d\rho'}{\int \mu_{\tilde{F}_{\rho'}}(\rho') d\rho'}. \qquad (27)$$

The amplitude of $\dot{E}(\mathbf{q}, \dot{\mathbf{q}})$ is regulated by the output ρ of the fuzzy control. This method is used to maintain the energy unchanged until the acrobot enters the attractive area.

4. Control in the attractive area

The attractive area is defined in (5). Its dynamics in this area is nonlinear, and a linear approximate model around the unstable straight-up equilibrium position is usually used for control. However, linearizing it by using only the position makes the attractive area very small. To achieve better control, the model in this area needs to be described more precisely.

Takagi and Sugeno have introduced model-based analytical method into fuzzy control [7]. This method gives us a more suitable way to describe the nonlinearity in the attractive area. The dynamic in the attractive area is captured by a set of fuzzy implications that characterize local relations. Then, a set of local controllers is designed based on the models using the parallel distributed compensation method. Finally, the fuzzy controller obtained by fuzzy blending of the local controllers is used to balance it. The stability of the fuzzy control system for balance control is guaranteed if a common symmetric positive definite matrix can be found for all local linear models.

4.1. Takagi-Sugeno fuzzy model

The Takagi-Sugeno fuzzy system is given by

Rule 1: If z is larger than c_1, Then $\dot{\mathbf{x}} = \mathbf{A_1 x} + \mathbf{B_1}\tau$,

Rule 2: If z is smaller than c_2, Then $\dot{\mathbf{x}} = \mathbf{A_2 x} + \mathbf{B_2}\tau$,

where $z = |q_1|/|q_2|$, c_1 and c_2 are constants, and $c_1 > c_2 > 0$.

It is clear that two linear models are used to describe the acrobot. In Rule 1, the acrobot is linearized with the coordinate $\mathbf{x_d} = [0\ \delta\ 0\ 0]^T$; and in Rule 2, it is linearized with the coordinate $\mathbf{x_q} = [0\ \theta\ 0\ 0]^T$ (where $\theta > \delta \geq 0$).

In the attractive area, $q_1 \in [-\lambda_1, \lambda_1]$ and $q_1 + q_2 \in [-\lambda_2, \lambda_2]$. Since λ_1 and λ_2 are very small, $\sin q_1$ and

$\sin(q_1+q_2)$ can be approximated by q_1 and q_1+q_2, respectively. According to equation (1) and (2), the linear approxi- mate model for the coordinate $x_\phi = [0\ \phi\ 0\ 0]^T$ (where $\mathbf{x}_\phi = \mathbf{x}_\delta$ or $\mathbf{x}_\theta$) is as follows:

$$\dot{\mathbf{x}} = \mathbf{A}(\phi)\mathbf{x} + \mathbf{B}(\phi)\tau , \qquad (28a)$$

where

$$\mathbf{A}(\phi) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31}(\phi) & a_{32}(\phi) & 0 & 0 \\ a_{41}(\phi) & a_{42}(\phi) & 0 & 0 \end{bmatrix}; \ \mathbf{B}(\phi) = \begin{bmatrix} 0 \\ 0 \\ b_3(\phi) \\ b_4(\phi) \end{bmatrix}, \qquad (28b)$$

$$q_\phi = \begin{bmatrix} 0 & \phi \end{bmatrix}^T,$$

$$\begin{bmatrix} a_{41}(\phi) & -a_{42}(\phi) & b_4(\phi) \\ -a_{31}(\phi) & a_{32}(\phi) & -b_3(\phi) \end{bmatrix} = \frac{\mathbf{M}(q_\phi)}{\det M(q_\phi)} \begin{bmatrix} -\beta & \beta & 1 \\ \alpha+\beta & -\beta & 0 \end{bmatrix},$$

$$\alpha = -(m_1 L_{g1} + m_2 L_1) g, \quad \beta = -m_2 L_{g2} g .$$

Substituting the coordinates x_δ and x_θ into (28b) yields the following two local linear models $(\mathbf{A}_1, \mathbf{B}_1) = (\mathbf{A}(\delta), \mathbf{B}(\delta))$ and $(\mathbf{A}_2, \mathbf{B}_2) = (\mathbf{A}(\theta), \mathbf{B}(\theta))$, respectively. So, the dynamics of the approximate fuzzy model is represented by

$$\dot{x} = \sum_{j=1}^{2} \mu_j(z)(A_j x + B_j \tau) / \sum_{j=1}^{2} \mu_j(z) , \qquad (29)$$

where $\mu_1(z)$ and $\mu_2(z)$ are the membership functions for Rules 1 and 2, respectively. They are defined as

$$\mu_1(z) = \begin{cases} 0; & 0 \le z \le c_2 \\ \dfrac{1}{2} + \dfrac{1}{2}\sin\dfrac{\pi}{c_1 - c_2}\left(z - \dfrac{c_1 + c_2}{2}\right); & c_2 < z \le c_1 \\ 1; & z > c_1 \end{cases} \qquad (30a)$$

$$\mu_2(z) = 1 - \mu_1(z) . \qquad (30b)$$

4.2. Design of model-based fuzzy controller

The concept of parallel distributed compensation ([14]) is utilized to design local controllers. The basic idea is to design a corresponding controller for each local linear model. This paper employs the pole assignment approach to design the local controllers. The full state is assumed to be available and the design results are given by

Rule 1: If z is larger than c_1 Then $\tau = -F_1 x$,

Rule 2 If z is smaller than c_2 Then $\tau = -F_2 x$.

Finally, the resulting overall fuzzy controller obtained by the fuzzy blending of the individual linear controllers is

$$\tau = -\sum_{j=1}^{2} \mu_j(z) F_j x / \sum_{j=1}^{2} \mu_j(z). \qquad (31)$$

This is used to balance the acrobot. Controller (31) is nonlinear in general. It is clear that the parallel distributed compensation method employs two controllers with automatic switching via fuzzy rules.

Substituting (31) into (29) yields the following fuzzy control system:

$$\dot{\mathbf{x}} = \frac{\sum_{j=1}^{2}\sum_{k=1}^{2} \mu_j(z)\mu_k(z)(A_j - B_j F_k)\mathbf{x}}{\sum_{j=1}^{2}\sum_{k=1}^{2} \mu_j(z)\mu_k(z)} . \qquad (32)$$

To guarantee stability, the result in [8] was applied to the fuzzy control system (32), and the following sufficient condition for stability was obtained.

Theorem 1: The fuzzy control system (32) is asymptotically stable at the unstable straight-up equilibrium position if there exists a common symmetric positive definite matrix P such that the following LMIs hold:

$$(\mathbf{A}_j - \mathbf{B}_j \mathbf{F}_k)^T \mathbf{P} + \mathbf{P}(\mathbf{A}_j - \mathbf{B}_j \mathbf{F}_k) < 0, \quad j, k = 1, 2 \qquad (33)$$

It is known that finding the matrix P is a convex feasibility problem. Now, this problem can be solved efficiently by using LMImethod[9] .

5. SIMULATION

The parameters of the acrobot are given as m_1=1kg, L_1=1m, L_{g1}=0.5m, I_1=0.083Nm^2, m_2=1kg, L_2=2m, L_{g2}=1m and I_2=0.33Nm^2. $\lambda_1 = \lambda_2 = \pi/4$ (rad/s) is used to divide the motion space. The parameters of the are c=2, k=1, E=24.5 J, Φ=15, c_1=4, c_2=0.1, δ=0 (rad/s) and $\theta = \pi/4$ (rad/s).

For the attractive area, substituting the parameters into (16b) yields two local linear models: (A_1, B_1) and (A_2, B_2). Two local controllers are designed by applying the method of parallel distributed compensation to (A_1, B_1) and (A_2, B_2). The local feedback gains F_1 and F_2 are determined by selecting (-2, –2.2, –2.4, –2.6) as the eigenvalues of the local linear subsystems. The overall parallel distributed compensation controller is

$$\tau = -\mu_1(z)\mathbf{F}_1\mathbf{x} - \mu_2(z)\mathbf{F}_2\mathbf{x} \ . \qquad (34)$$

A symmetric positive definite matrix P is obtained by using the LMI algorithm. So, the fuzzy control system is asymptotically stable for fuzzy control law (24).

Figure 4 shows simulation results for the initial condition $\mathbf{x}(0) = [\pi\ \ 0\ \ 0\ \ 0]^T$. When $0 \le t < 0.54$ s, the model-free fuzzy controller is used until the energy reaches E. When $0.54 \le t < 2.5$ s, the fuzzy sliding-mode controller is used to move the acrobot into the attractive area. When $t \ge 2.5$ s, the model-based fuzzy controller is used for balancing control. The simulation results show that the response is soft when the control law changes, and the state converges smoothly to the unstable straight-up equilibrium position.

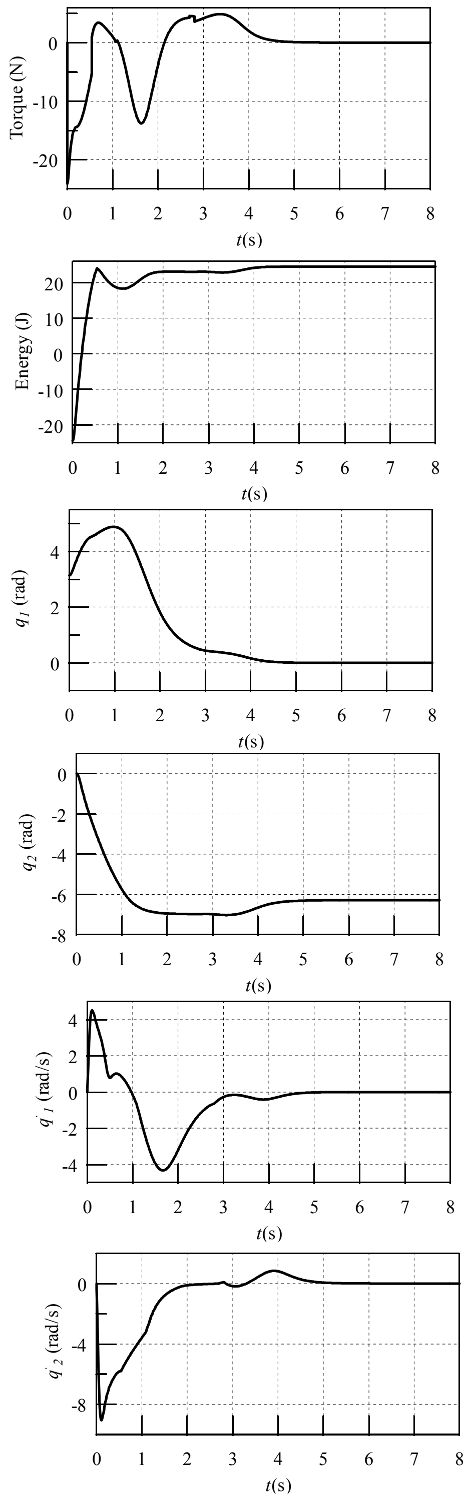


Figure 4. Simulation results.

6. Conclusions

A control strategy combining model-free fuzzy control, a fuzzy sliding-mode control and model-based fuzzy control has been developed for controlling an acrobot. The model-free fuzzy controller is used for swing-up control. It is designed to guarantee that the energy of the acrobot increases with each swing, and the amplitude of the control torque decreases as the energy increases. A fuzzy sliding-mode controller is employed to control the second link. It drives the angle of the second link towards zero, and maintains the energy almost unchanged. The model-based fuzzy controller is used for balance control and is designed by combining the Takagi-Sugeno fuzzy model with the method of parallel distributed compensation. The stability of the fuzzy control system for balance control is guaranteed by a common symmetric positive matrix. Simulation results have demonstrated the validity of the method. This strategy ensures that the acrobot enters the attractive area quickly, and overcomes the difficulty of the capture of the acrobot in the swing-up ares. It guarantees that the acrobot enters the attractive area very fast.

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